2. SEQUENCY FILTERS FOR TIME AND SPACE SIGNALS

2.1 Correlation Filters for Time Signals

2.1.1 Generation of Time Variable Walsh Functions

Figure 52 shows a circuit for the generation of periodically repeated Walsh functions \( \text{cal}(i, \theta) \) and \( \text{sal}(i, \theta) \). This circuit is based on the multiplication theorem of the functions \( \text{wal}(j, \theta) \) as given by Eq. (1.1.4-3).

Fig. 52. Generator for time variable Walsh functions using products of Rademacher functions. B, binary counter stage; \( \times \), multiplier = exclusive OR-gate; \( z \), input for trigger pulses; \( n \), input for reset pulses.
2.1.1 Generation of Time Variable Walsh Functions

Binary counters B1 to B4 produce the functions \( \text{wal}(1, \theta) = \text{sal}(1, \theta) \), \( \text{wal}(3, \theta) = \text{sal}(2, \theta) \), \( \text{wal}(7, \theta) = \text{sal}(4, \theta) \) and \( \text{wal}(15, \theta) = \text{sal}(8, \theta) \). The multipliers shown in Fig. 52 produce from these Rademacher functions the complete system of Walsh functions. \( \text{wal}(0, \theta) \) is a constant positive voltage.

The multipliers are gates having a truth table as shown in Table 4, since Walsh functions assume the values +1 and -1 only. Comparison of this truth table with that of the exclusive OR-gate or the half adder shows that the multipliers in Fig. 52 may be exclusive OR-gates, if an output 0 stands for a positive voltage +\( V \) and an output 1 for a negative voltage -\( V \).

Table 4. Truth tables for a multiplier for two Walsh functions (a) and for an exclusive OR-gate (b).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>-1</td>
<td>0 1</td>
</tr>
<tr>
<td>+1</td>
<td>+1</td>
<td>0 1</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>1 0</td>
</tr>
</tbody>
</table>

Consider a Walsh function generator having 20 binary counters rather than 4 as shown in Fig. 52. A total of \( 2^{20} = 1048576 \) different Walsh functions can be obtained. Nineteen exclusive OR-gates are required to produce any one of the \( 20^{20} \) possible functions. The accuracy of their sequency will depend on the trigger pulse generator which drives the binary counters. There are no drift or aging problems. It is worthwhile to compare the simplicity of such a generator with that of a frequency synthesizer delivering a million discrete sine functions. On the other hand, representative switching times of the fastest digital circuits are presently between 100 ps and 10 ns. This restricts the highest sequency of Walsh functions from \( 10^8 \) zps = 100 Mzps to \( 10^{10} \) zps = 10 Gzps at the present time. Sine waves with frequencies of 100 MHz to 10 GHz were produced decades ago.

The generator of Fig. 52 is not well suited for high switching speeds because of the accumulative delay in the exclusive OR-gates. The build-up of delays is avoided by the circuit shown in Fig. 53 [1]. There are again four counter stages B that produce the Rademacher functions \( \text{wal}(1111, \theta) \) to \( \text{wal}(1, \theta) \). They are parallel triggered for faster operation. The Rademacher functions are differentiated and yield the trigger pulses \( \text{wal}'(1111, \theta) \) to \( \text{wal}'(1, \theta) \) as shown in the pulse diagram of Fig. 54. The obtained negative trigger pulses\(^1\) \( \text{tri}(1000, \theta) \) to \( \text{tri}(1, \theta) \) may or may not pass through

---

\(^1\) The differentiated Rademacher functions do not have to be rectified to obtain these trigger pulses, since the following AND-gates will automatically suppress positive pulses.