Neuronic Equations Revisited and Completely Solved

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1 Introduction

Since my first meeting with Cybernetics, it has been crystal clear to me that its true object is the study of "intelligence" (calling it "natural" or "artificial" is misleading and causes only confusion). Can a physicist, with his mind and tools, have a say in such matter? Although today the question easily receives brazenly affirmative answers (I fear that too many problems lay ahead still totally ignored), at the time of my first endeavours the situation stood quite differently. This was lucky, in that little bias could then poison a young, enquiring mind; luckier perhaps than today, when so many claim problems to be "almost solved" and pour down answers, when even questions cannot yet be soundly formulated. For this reason I shall refrain from sweeping statements and limit my discussion to only one of the three basic elements of my model (see below): a set of nonlinear equations that describe the behaviour of a system of coupled binary decision elements in discrete time ("neuronic equations": NE). Their solution is in any case an essential preliminary to that of the remaining parts, "mnemonic equations" and "adiabatic learning hypothesis", which are strongly connected to anatomical, or technological, structural information.

I shall only mention some points which appear to me now, after a quarter century of meditation, of special relevance, summing them up in few sentences:

1. Strategy: the study of neural models, or organization, or complex structured systems (natural languages, social structures ...) needs a common approach as regards functional and relational aspects.
2. Levels of a hierarchical nature are essential in this study; each level requires its specific logic, mathematics, relation to other levels; their discrete quantization is basic for the stability of the system.

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3. Quantitative information about populations of levels, also clearly basic, cannot come at the present time from brain research alone (I have turned to natural languages and all sorts of other systems for it, but this is another story).

Many situations are met which are familiar to a physicist: coarse graining, Wilson's renormalization group, spin-like systems are "natural" to both fields. There is however a crucial difference: a system structured into discrete levels needs additional care, as limiting processes may destroy the very structure one tries to understand. Neuronic equations describe a level of activity; their language, with respect to higher levels, is like that of micro- to various scales of "macro"-physical objects. A "neuron" in them is just a decision element; no one-to-one correspondence is assumed with biological neurons. Nevertheless, there is an interesting connection: by enlarging (if and as needed) the number of "neurons" in our equations, their solutions, which I shall exhibit in (formally!) compact and explicit form, exhaust all behaviours one can possibly expect from any nonlinear system presented in discretized form; this includes dynamics, cycles, chaos, transitions, etc.

Most of present day mathematics, as applied to the physical or other sciences, originates historically from the study of the "continuum": gravitation, fields, etc.; it carries with it an underlying notion of "space", be it Descartes's emptiness or the physicist's "vacuum", into which things happen whose description is to be sought by exploring "neighborhoods", through differential, i.e., linear approximations. Higher order, nonlinear terms are added as corrections when the linear description of reality becomes poor.

Everybody knows, of course, that this is done for lack of techniques apt to solve in a general and exact way even trivial nonlinear equations; the cases in which this is possible are known by name in the literature, and mostly belie the guesses that stem from such "linearizing" Weltanschauung. We find thus two concomitant elements: an historical propensity to "think linear"; the hard fact that the only general method available for computation is to fragment a problem into linear pieces.

There is also, however, an ever increasing amount of instances in which nonlinearity is basic: all that goes with boolean algebra, computer sciences, decision-making, models of neural activity, etc.. This is an entirely different sort of universe for the student of Nature, with a correspondingly different Weltanschauung. Also here, things become as clumsy (though elegant verbiage may act as a cover) as they frankly appear to a physicist whose expansions refuse to converge. The long-term behaviour, the collective actions of aggregates of interconnected yes-or-no decision elements, and many other such questions, appear unanswerable; or at least, the effort is not made.

Both these aspects are present in the model of neural activity which we proposed (Caianiello 1961) in order to extend and algebrize the pioneering work