Lecture 28  Path Calculation of F

Wright (1922) originally derived his system for calculating individual inbreeding coefficients from his general method of path coefficients, or standardized partial regression coefficients (Wright 1934). As used in calculating inbreeding, paths point out the flow of identical genes, and the coefficients are identity probabilities. The path coefficient method constitutes a geometric approach to analysis, as opposed to the algebraic approach of Fisherian statistics. Path coefficients are still employed for many analytical purposes, especially in human genetics (Li 1975) and the social sciences, although somewhat out of favor in quantitative genetics.

The inbreeding coefficient is the probability that an individual receives identical genes from its two parents. Obviously, this probability is 0 unless the parents have one or more ancestors in common. Consider the pedigree shown in Fig. 28.1. We wish to calculate $F_J$, the inbreeding coefficient of individual J. G and H, the parents of J, have a common ancestor, A; we assume that the members of the pedigree are unrelated except as shown by the pedigree. $F_J$, then, is the probability that J receives simultaneously from G and H identical genes originating in A.

Let A's genotype be symbolized as $X_A^A$. The probability that A transmits $X_A$ to C is then 1/2. If this occurs, the probability that C transmits $X_A$ to its offspring G is also 1/2; and the probability that, in that event, G transmits $X_A$ to J, 1/2. The probability that J receives $X_A$ from A via C and G is the joint probability of all three of these events, $(1/2)^3$.

The three steps, from A to C, from C to G, and from G to J, form a path. The coefficient associated with each step in the path is the probability that $X_A$ passes through that step, and the coefficient of the whole path is the product of the individual coefficients for each step.

Fig. 28.1. Pedigree for the path calculation of inbreeding (Falconer 1960). All individuals can be assumed to be noninbred and unrelated except as shown in the pedigree.
It is also possible for \( X_A \) to pass from A to J by way of D, F, and H. This is a path of four steps, each with coefficient 1/2, so that the probability that J receives \( X_A \) from A via D, F, H is \((1/2)^4\).

If these two paths are realized simultaneously, J will have genotype \( X_A X_A \). The probability that both paths will be realized is the product of their separate probabilities:

\[
G(J: X_A X_A) = (1/2)^3(1/2)^4 = (1/2)^{3+4}.
\]

Individual A can also transmit gene \( X_A^* \) to J via the same two paths, so that

\[
G(J: X_A X_A^*) = (1/2)^{3+4}.
\]

Therefore, the probability that J receives two copies of the same gene in A, via C, G and via D, F, H is

\[
G(J: X_A X_A \text{ or } X_A^* X_A^*) = 2(1/2)^{3+4} = (1/2)^{3+4-1}.
\]

J can receive \( X_A \) from A via C, G, and at the same time receive \( X_A^* \) from A via D, F, H. The probability of this event is again \((1/2)^{3+4}\). Because J can also receive \( X_A^* \) via C, G and \( X_A \) via C, D, F with the same probability,

\[
G(J: X_A X_A^*) = (1/2)^{3+4-1}.
\]

But \( X_A \) and \( X_A^* \) can be identical, with probability \( F_A \). Therefore, the probability that J receives identical genes \( X_A \) and \( X_A^* \) from A is

\[
G(J: X_A X_A^*) I(X_A, X_A^*) = (1/2)^{3+4-1} F_A.
\]

Finally, therefore, the probability that I receives identical genes from A is the inbreeding coefficient of J,

\[
F_J = (1 + F_A)(1/2)^{3+4-1}.
\]

The sum

\[
G(J: X_A X_A) + G(J: X_A X_A^*) + G(J: X_A X_A^*) = 2(1/2)^6 = 1/32
\]

is less than 1. It is not a certain event that J will receive a gene from A by either path. For example, C might transmit to G the gene it received from its other parent, rather than the one it received from A. Then J cannot receive a gene from A via the path A-C-G-J. Every individual in either path can similarly fail to transmit an A gene.

In general, if the two parents of an individual have a common ancestor, the probability that the individual will receive identical genes originating in that ancestor from both parents will be

\[
F_J = (1 + F_A)(1/2)^{n+m-1},
\]

where J is the individual, A the ancestor common to both of J’s parents, and the paths from A to J have \( n \) steps going through one of J’s parents and \( m \) steps going through the other.

The pedigree in Fig. 28.1 is a relatively simple one. It is, however, only part of a more complex pedigree, which we have already presented in Fig. 27.2. It is