7 Infinite Dimensional Formulations of Some Dynamic Traffic Assignment Models

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7.1 Introduction

Traffic assignment models attempt to determine the usage of each route and/or link in a transportation network, given information about the number of trips being taken between various locations, the characteristics of the network, and the characteristics of the vehicles on the network. Though the term “assignment” seems to connote a prescriptive process in which vehicles are assigned to particular routes, there are both descriptive/positive and prescriptive/normative traffic assignment models.

Obviously, these models are of practical significance because of congestion. If transportation networks were not congested, there would be little practical interest in traffic assignment models. As it turns out, this is also what makes these models interesting from a theoretical standpoint. Indeed, as observed by Wardrop (1952), the route choices of drivers both cause and are influenced by congestion. This feedback process is illustrated in Fig. 7.1 and has been modeled in two ways. One approach is to ignore the adjustment mechanism and focus exclusively on the resulting equilibrium (see, for example, Beckmann et al., 1956; Smith 1979; Dafermos, 1980). The other approach is to explicitly model the day-to-day adjustment process (see, for example, Smith, 1979; Cascetta, 1991; Dupuis and Nagurney, 1993; Friesz et al., 1994).

In spite of the importance of congestion in traffic assignment, traditional traffic assignment techniques use very simple models of congestion. In fact, they completely ignore the inherently dynamic nature of congestion. That is, they employ link performance functions which describe the relationship between the average number of vehicles using a link during a period, and the average travel time experienced by those vehicles during that period (see, for example, Branston, 1976, or Boardman and Lave, 1977). In addition, in keeping with their use of aggregate performance functions, they ignore departure-time choices and the relationship between those choices and travel-times. Finally, they ignore how the route and departure-time choices of travelers change in the short-term (say, in response to congestion, information, or guidance).

One way of incorporating the dynamics of congestion is illustrated in Fig. 7.2. Unlike the traditional paradigm, this new paradigm distinguishes between day-to-day adjustment behavior and within-day adjustment behavior. Perhaps more importantly, this new paradigm explicitly considers the timing of trips (i.e., the trajectory of route choices) and how/when different vehicles interact. That is, the dynamics of
congestion formation are considered as are the impacts of time-varying travel costs on behavior.

Though no complete model of this kind exists, some of the pieces have been considered. First, a variety of different models of the (hopefully) resulting equilibrium have been developed, some of which include only route choices and some of which include both route and departure-time choices (see, for example, Smith and Ghali, 1990; Drissi-Kaitouni, 1990; Cascetta, 1991; Janson (1991); Friesz et al., 1993; Ran et al., 1993; and Bernstein et al., 1993). They are perhaps best viewed as models of default (or usual) behavior and are most appropriate for transportation planning applications.

In addition, there have been several attempts to model the adjustments that drivers make to their routes (and departure-times) in response to changing conditions (see, for example, Chang and Mahmassani, 1988; and Ben-Akiva et al., 1991). They are perhaps best viewed as pre-trip or en-route adjustment models and are most appropriate for traffic control applications.

Where the developers of these dynamic traffic assignment models seem to have had the most difficulty is in the development of the physical model of traffic flow/dynamics. While it is relatively easy to specify individual link performance functions, network effects complicate things dramatically. For example, consider the deterministic queueing approach introduced by Vickrey (1969). Though it is quite easy to model the travel time for a vehicle entering a link at a specific time as a function of the number of other vehicles in the queue at the tail of the link at that time, it is very difficult to model the same travel time as a function of the departure patterns over the entire network (i.e., incorporating multiple origin-destination pairs, paths, and departure-times).

Two broad approaches have been suggested to replace the closed-form perform-

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1 Indeed, early dynamic traffic assignment models took an entirely different approach, using link exit-rate functions (see, for example, Merchant and Nemhauser, 1978a;1978b; Carey and Srinivasan, 1988; and Friesz et al., 1989). However, this approach, in addition to being somewhat less intuitive, has some problems which make it unworkable in many cases.