Chapter 12  Introduction to Welfare Economics and General Equilibrium Analysis

1. Introduction

In the earlier chapters we dealt with the behavior of individual agents be they consumers or producers. We also dealt with markets for individual goods. In this chapter we discuss the simultaneous behavior of all agents. The discussion attempts to clarify the relationship between what is “good” for the group and what is “good” for its individual constituents. When we speak of this relationship we are in the realm of welfare economics. This will be done in Section 2. In our discussion we will assume that “good for the society” means Pareto efficient. What is “good for individuals” will mean a general competitive equilibrium. In the absence of externalities, in addition to some other less striking assumptions, it is shown that the two “good” coincide. We call attention to the shallowness of the philosophic import of this result which may be stated as follows: If every member of society is an island into themselves, then what is best for society coincides with what is best for the individual. (How else can it be?)

In Section 3 we discuss the question of existence of competitive equilibrium. In both sections we assume that preferences are representable by utility functions which are continuously differentiable up to and including the second order. We also assume that technology sets are described by equally smooth production functions.

2. Production, Pareto Efficiency, and Competitive Equilibrium

Let \( I = \{1, 2, 3, \ldots, n\} \), \( J = \{1, \ldots, m\} \), \( L = \{1, 2, \ldots, l\} \) be the index sets of consumption agents, goods and firms respectively. The preferences of consumer \( i \) are represented by \( u^i: X_i \rightarrow \mathbb{R}^+ \), where \( X_i \) is the consumption set of consumer \( i \), for \( i \in I \). We shall think of \( X_i \) as \( \mathbb{R}^m_+ \). In stating that \( u^i \) depends only on \( x_i \) we already have assumed away externalities in consumption.

Let the technology of firm \( r \) be given by:

\[
Y_r = \{ y_r \in \mathbb{R}^m \mid f^r(y_r) \leq 0 \}, \quad r \in L,
\]

where \( y_r \) is a netput vector, i.e., a vector whose components are differences between outputs and inputs. We are assuming away externalities in production as well. Let \( \omega_{ij} \) be the initial endowment of consumer \( i \) in good \( j \) and let \( \theta_{ir} \) be the share of consumer \( i \) in the profits of firm \( r \). Such profits, \( \pi_{ir} \), are given by \( \pi_{ir} = py_r \) if \( p \) is a price vector and \( y_r \) is a netput vector.
a) Efficiency in Production

Let \( y = \sum_r y_r \) be a given aggregate netput vector. We say that \( y \) is feasible if \( f'(y_r) \leq 0 \) so each individual firm's contribution is technologically feasible and if \( y + \sum_i \omega_i \geq 0 \), where \( \omega_i \) is the initial endowment vector for consumer \( i \). We say that \( \hat{y} \) is an efficient aggregate production (EAP) if it is feasible and if there does not exist another feasible \( y \) such that \( y \succeq \hat{y} \) in the sense of vector ordering introduced earlier. Thus, \( \hat{y} \) is EAP if it can't be improved upon by producing more output or utilizing less input since \( y \succeq \hat{y} \) means \( y_j \geq \hat{y}_j \) for all \( j \) with strict inequality for at least one \( j \) and since \( y_j \) is negative if \( j \) is a net input and non-negative if \( j \) is a net output.

Now suppose \( \hat{y} \) is EAP then, by the scalerization lemma*, it solves the following mathematical programming problem:

\[
\text{maximize } \sum_j \alpha_j \sum_r y_{rj} \text{ subject to: } \\
- f'(y_r) \geq 0, \quad r \in L \\
\sum_r y_r + \sum_i \omega_i \geq 0,
\]

where \( \alpha_i > 0 \) for all \( j \in J \).

By the first order necessary conditions we conclude the existence of multipliers \( \nu_r \) and \( \lambda_j \) that:

(i) \( \alpha_j - \nu_r \hat{f}_r^j + \lambda_j = 0, \quad r \in L, \quad j \in J \)

(ii) \( \nu_r \hat{f}_r = 0, \quad \mu \cdot \left( \sum_r y_r + \sum_i \omega_i \right) = 0 \)

(iii) \( \nu_r \geq 0, \quad \mu \geq 0 \),

where \( \hat{f}_r^j = \frac{\partial f'(y_j)}{\partial y_{sj}} \middle|_{y_j = \hat{y}_j} \).

But (i) implies that \( \nu_r > 0 \) provided \( \hat{f}_r^j \neq 0 \), i.e., provided the vector \( (\hat{f}_r^1, \ldots, \hat{f}_r^m) \neq 0 \). This condition is easily satisfied, think of the case where \( f'(y_r) = y_r^1 - g(y_r^2, \ldots, y_r^m) \). Then we would have, by (ii), \( \hat{f}_r(\hat{y}_r) = 0 \). Thus if \( \hat{y} \) is AEP then its constituent netput vectors are on the frontiers of the technology sets of the firms provided these sets are nontrivial.

Suppose furthermore that \( Y_r \) are convex sets; then we have:

(iv) \( \sum_j \alpha_j \sum_s \hat{y}_{sj} + \sum_j \mu_j \left( \sum_r \hat{y}_{rj} - \sum_i \omega_{ij} \right) \geq \sum_j \alpha_j \sum_r y_{rj} + \sum_j \mu_j \left( \sum_r y_{rj} - \sum_i \omega_{ij} \right) \) for all \( y_r \in Y_r, \ r \in L \).

Inequality (iv) holds in particular if we set \( y_r = \hat{y}_r \) for \( r \neq r_0 \). Thus we have

(v) \( \sum_j (\alpha_j + \mu_j) \hat{y}_{r_0j} \geq \sum_j (\alpha_j + \mu_j) y_{r_0j} \) for all \( y_{r_0j} \in Y_{r_0} \).

* See remark following Theorem 5.10 in mathematical appendix.