Mean Field Theory for Spin Glasses

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In recent years much progress has been made in this field, and after a great deal of effort self-consistent mean field approximation has been obtained [1] (sometimes this construction goes under the name of broken replica theory). This theory should be exact for weak long-range forces or when the dimensions of the space become very large (fluctuations are neglected). The ideas involved in this construction are rather different from those of the mean field approximation for other models and take care of the peculiar properties of spin glasses. In this paper we will not show how these ideas have been developed. We will also skip most of the technical details and concentrate our attention on a few important physical results.

A spin glass is a disordered magnet; in a typical example 50% of the randomly chosen bonds between spin pairs are ferromagnetic while the others are antiferromagnetic. From the experimental point of view [2], spin glasses are characterized by the presence of a low-temperature phase in which the relaxation time for attaining equilibrium is very large, as is the case in real glasses. The magnetic susceptibility in the presence of a time dependent magnetic field depends on the frequency even in the region of a few Hertz and the power spectrum of the magnetization behaves as the inverse of the frequency (1/f noise).

From the theoretical point of view, these properties may be connected to the practical impossibility of finding numerically the ground state of a spin glass, if the number of spins is not very small; indeed, according to conventional wisdom, the number of operations which any algorithm needs to find the ground state increases exponentially with the volume (the problem is NP complete in more than 2 dimensions). There is an exponentially large number of spin configurations which are local ground states, in the sense that their energy increases if only one spin is flipped. It is very easy to find efficient algorithms for finding local minima. In contrast, it is very difficult for the algorithm to find the global minimum, i.e. the true ground state.

Exhaustive numerical simulations on small systems show that different minima (of the energy at zero temperature or of the free energy at finite temperature) are macroscopically different and that the energy difference between macroscopically different minima remains of $O(1)$ even when the volume goes to
infinity. This situation is rather unusual in statistical mechanics; macroscopically different configurations with the same total free energy are normally present at the point where a first order phase transition happens (which is an isolated point). For spin glasses this phenomenon is present in a large range of temperatures and magnetic fields where no first order phase transition is seen from the thermodynamical point of view. A strong effort has been needed to find the appropriate theoretical framework to study these systems.

The origin of this peculiar behaviour is the presence of frustration; it is not possible that all pairs of spins which are connected by a ferromagnetic (antiferromagnetic) bond have the same (the opposite) sign. Different terms in the Hamiltonian push in different directions and the number of possible compromises is very large. For example in the Ising case the Hamiltonian is

$$H = -\frac{1}{2} \sum_{i,k} J_{i,k} \sigma_i \sigma_k - h \sum_i \sigma_i, \quad (1)$$

where $h$ is the magnetic field and the $J$'s are the coupling between the spins. It is clear that if the product $J_{12}J_{23}J_{31}$ is negative, it is impossible to find configurations of the $\sigma$'s such that $\sigma_1 J_{12} \sigma_2$, $\sigma_2 J_{23} \sigma_3$, $\sigma_3 J_{31} \sigma_1$ are all positive.

There are many systems in which different terms in the Hamiltonian, or different constraints, are in competition with one another. This happens very frequently when the system is complex. Typical examples are real glasses, other NP complete problems like the travelling salesman or the matching problem, protein folding, biological organization, prebiotic evolution, neuron network and so on. At present, many of the concepts that have been developed in the study of spin glasses, are also becoming useful for these systems, and it is quite possible that, in the long run, the applications of these ideas beyond solid state physics will be the most interesting ones. As we shall see later, the characteristic feature of spin glasses in the low temperature phase is the existence of many stable equilibrium states, which differ microscopically, but not macroscopically, from one another. The theory of these systems will not be simple [3], because it must reflect the complexity of the problem we face.

Suppose that we have a system whose Hamiltonian $H_J[\sigma]$ depends on the configuration $[\sigma]$ of the system and on some control variables $J$, which are distributed according to a probability distribution $P(J)$. For each choice of the $J$'s we can compute the partition function

$$Z_J = \sum_{\{\sigma\}} \exp \left\{ -\beta H_J[\sigma] \right\} \quad (2)$$

and free energy density...