An Asymptotic Theory for a System of Weakly Nonlinear Wave Equations

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1. Introduction

In this paper an asymptotic theory for a class of initial-boundary value problems for systems of nonlinearly coupled wave equations is presented. The theory implies the well-posedness of the problem in the classical sense and the validity of formal approximations on long time-scales. As an application of the theory an initial-boundary value problem for a system of nonlinearly coupled wave equations is studied using a two-timescales perturbation method. From an aero-elastic analysis it is shown that this initial-boundary value problem may be regarded as a model describing the galloping oscillations of overhead transmission lines.


Galloping can be defined as a low frequency, large amplitude phenomenon involving an almost purely vertical oscillation of single conductor lines on which for instance ice has accreted. The frequencies involved are so low that the assumption can be made that the aerodynamic forces are as in steady flow. Another consequence of these low frequencies is that structural damping may be neglected. In severe cases galloping may give rise to conductor damage due to impact of conductor lines. The usual conditions causing galloping (see also [6]) are those of incipient icing in a stable atmospheric environment implying uniform (but not necessarily high velocity) airflows.

Recently in [1] an oscillator with two degrees of freedom has been considered to describe the oscillations of a rigid circular cylinder with a small ice ridge in a uniform airflow. In that approach a system of two coupled, ordinary differential equations is obtained, describing the displacements of the cylinder in two directions. And in [4] a cylinder-shaped transmission line has been considered to describe the vertical displacement of the conductor due to galloping. In this paper the vertical as well as the horizontal displacements of a circular cylinder-shaped conductor with a small ice ridge will be considered in a horizontal airflow. In order to describe the galloping oscillations a right-handed coordinate system is set up where one of the endpoints of the conductor is the origin. Through this point three mutually perpendicular axes
(the $x$-, $y$-, and $z$-axis) are drawn, where the $z$-axis coincides with the direction of gravity. The endpoints of the conductor are supposed to be in $(0,0,0)$ and $(1,0,0)$, where $l$ is the distance between the pylons. To model galloping a cross-section (perpendicular to the $x$-axis) of the conductor is considered. It is assumed that all cross-sectional shapes are identical and symmetric. Along the axis of symmetry of a cross-section a vector $e_s$ is defined to be directing away from the ice ridge and starting in the centre of the cross-section. In fig. 2.1 the centre of the cross-section is considered to be at $x = x_0$, $y = y_0$ and $z = z_0$ with $0 < x_0 < 1$.

Let $v(x_0,t)$ respectively $w(x_0,t)$ denote the $y$-coordinate respectively the $z$-coordinate of the centre of the cross-section at $x = x_0$ and time $t$. It is assumed that every cross-section perpendicular to the $x$-axis oscillates in a plane parallel to the $(y,z)$-plane. Furthermore, the torsion of the conductor is not taken into account. Let the static angle of attack $\alpha_s$ (assumed to be constant and identical for all cross-sections) be the angle between $e_s$ and the uniform airflow $v_\infty$. In this uniform airflow with flow velocity $v_\infty = v_\infty e_y (v_\infty > 0)$ the conductor may oscillate due to the lift force $Le_L$ and the drag force $De_D$. It should be noted that the drag force $De_D$ has the direction of the virtual wind-velocity $v_s$ and that the lift force $Le_L$ has a direction perpendicular to $v_s$. Now the conductor is considered to be a one-dimensional continuum in which the only interaction between different parts is a tension $T$, which for all times is assumed to be constant along the conductor. The equations describing the horizontal and the vertical motion of the conductor are given by