New Developments in Quantum Field Theory

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Abstract. These notes are a short summary of a lecture presented at the XXIV International Conference on High Energy Physics in Munich. As an example of an interesting new development in quantum field theory, two-dimensional conformal field theory is discussed in somewhat more detail. It is argued that two-dimensional conformal field theory is, in essence, representation theory of a pair of "chiral algebras" and it is indicated how the basic objects of such a theory can be obtained from certain representation-theoretic constructs.

Introduction. While my lecture was a broad review of recent (and some less recent, but important) developments in quantum field theory, these notes are limited, in essence, to one chapter of my review, dealing with a particularly active and vital domain of recent activities: two-dimensional conformal field theory. This does not imply any value judgment on other areas in quantum field theory which have seen exciting recent developments, such as the perturbative and non-perturbative exploration of the standard model, non-perturbative aspects of QCD, renormalization group constructions of renormalizable and (perturbatively) non-renormalizable quantum field models, local and global anomalies, applications of quantum field theory to pure mathematics, etc. In fact, I may be choosing the chapter that is least important to serious and honest particle physicists. But it happens to be the one I am personally somewhat involved in, researchwise, at the present time, and it lends itself fairly well to a reasonably elegant presentation. I should add that I find other developments, such as the non-perturbative control of non-abelian gauge theories, e.g. pure Yang-Mills in four dimensions, or of models like the Gross-Neveu model in $2+\epsilon$ dimensions, with $0 \leq \epsilon < 1$, at high energies more important for our understanding of relativistic quantum physics, and topological quantum field theory is, perhaps, more important for our understanding of some problems in pure mathematics than conformal field theory. But those topics, in particular the ones centered around renormalization group constructions are far too complicated to lend themselves to a meaningful review on a few pages. I have a (perhaps somewhat vague) plan to write an extensive review on quantum field theory with many references for a review journal. Until then, the reader may forgive me for having selected the chapter on conformal field theory! I should like to emphasize that whatever may look new to the reader in my presentation of two-dimensional conformal field
theory is based on joint work with Giovanni Felder and Georg Keller whom I thank for everything they have taught me.

Of course, the reader has the right to know, in the form of a list of contents and a few remarks, what my lecture was about. In order not to do injustice to too many people, and because I am short of time for a painful job, I must refrain from presenting an extensive list of references.

I shall then start with my mini-summary of quantum field theory.

I. What is quantum field theory?
(1) Quantum field theory (QFT), as local, relativistic QFT, is a basic theoretical framework for the physics of the fundamental interactions (electro-weak and strong), excluding gravitation, between elementary particles in an energy range well below the Planck mass (~10^{19} GeV). In this energy range the (quantum-?) dynamical aspects of space-time can probably be neglected, space-time is treated as static and classical, and, in developing local, relativistic QFT, one attempts to combine the fundamental principles of relativity theory (finiteness of signal propagation speed, space-time symmetries) and of quantum mechanics, in the form of a mathematically consistent, or, at least, "cutoff-independent" gauge theory. These attempts have been immensely successful, phenomenologically, but, theoretically, many open questions remain - inspite of enormous progress.

(2) More generally, QFT can be viewed as an incarnation of the quantum mechanics of systems with infinitely many degrees of freedom. From this point of view, it is a basic theoretical framework also for many-body theory; an area which has seen many important and exciting recent developments.

(3) In the form of Euclidean field theory, QFT is the theoretical framework for a quantitative theory of critical phenomena in statistical mechanics. This aspect of QFT leads naturally to the study of conformal field theory which I shall review in some more detail. Techniques developed in the context of "Euclidean field theory and critical phenomena" have found useful applications in other areas, e.g. classical dynamics.

(4) QFT is a source of inspiration and technique and sometimes a good guide in many areas of pure mathematics, in particular in probability theory, operator algebras, commutative and non-commutative geometry and topology; (one might think of the work of von Neumann, Connes, and many others; of constructive quantum field theory, and of all the developments triggered by the work of Witten and his followers).

II. Dominant features of recent progress in QFT
Here is a sketch of my view of areas in QFT where real progress has been made, during the past few years.

(1) Understanding non-perturbative renormalization of perturbatively renormalizable and non-renormalizable quantum field models with an ultraviolet stable renormalization