PARAMETER ESTIMATION IN SIGNAL PROCESSING

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ABSTRACT: The filtering problem encountered in signal processing uses implicitly a model of the signal generator. The simultaneous estimation of the parameters of this model and the signal can be solved by the partitioning approach if there exist optimal, or at least sub-optimal filters for all possible values of the unknown parameters. We develop here this approach for the case of a Gauss-Markov signal observed through a counting Poisson process, problem that commonly arises with sensors using radioactive tracers.

I- INTRODUCTION

The so called "Filtering problem" consists on the construction of a process $X(t)$ that represents, in the best possible way, another unobserved process $x(t)$, using all the possible observations $\{y(t), \forall t\}$. In a very general form we can describe this situation as a search for the best attainable model of process $x(t)$. Of course in most of the classical situations in signal processing something is known about the real mathematical model that generates the signal $y(t)$, but this knowledge is either imprecise, or corrupted by other processes called noise processes. Stochastic processes theory gives an interesting and useful framework for mathematical formulation of those concepts.

Figure 1
As represented in figure 1 the filtered process has to be generated by a MODEL of the GENERATOR, where p stands for the "true" parameters of the generator and P is an estimator of p that parametrizes the model.

It can be objected that most of the classical signal processing results (Wiener, 1949), (Kalman-Bucy, 1961), are linear filters and they do not explicitley modelize and estimate parameters of models, but they directly estimate process values; two remarks must be done to understand their particular apparent straightforward result: those linear filter results assume a very precise knowledge of the generator, and moreover this generator is linear and accepts all extrapolations in a very simple way. As soon as the linear models are variable or unknown, the well known "adaptative" signal processing becomes a nonlinear problem and as it was shown in (Lainiotis, 1971) an optimal solution is the so called "partitioning" that separates the parameter estimation problem from the filtering problem. This approach assumes that if the true values of parameters were known, the filtering would be perfect, or at least satisfactorily performing following the model shown in fig. 1, therefore this approach can give optimal results for all the situations when this happens. It is also necessary to assume that it is possible to build as many perfect filters as possible values of the parameter, or in practice at least to cover by means of interpolation all the possible points of the parametric space. Although the requisites of the optimal "partitioning method" are seldom fully satisfied, approximate versions give practical acceptable results and, in favorable situations, they accept theorical calculation of error bounds, or at least convergence properties.

II- PROBLEM STATEMENT

Our purpose is to fully developp a very important case where this parameter estimation technique gives useful results, its importance comes from the general measurement of the intensity, or mean rate, of point processes in sensors based in electron or photon counting, as is the case of the measurement of flows through radiactive tracers, of a classical use in medecine and biology.

As an example we shall briefly describe in Figure 2 a practical situation where this situation occurs.