3. Signal Processing Fundamentals

In this chapter, we shall review material relevant to the understanding, design and application of optical and other devices discussed in this book. Signals can be analog, discrete or digital. For the analog case, the time or space variable is continuous. For the discrete case, the time axis is sampled at a fixed interval or for discrete values, even though the amplitude remains analog or continuous. Of course, for a digital system, the time is discrete and the amplitude is represented by a digital number. The spatial signal can also be two- or multidimensional.

Sections 3.1 and 3.2 discuss analog and discrete one-dimensional systems, respectively. The concepts of Fourier and z-transform, impulse and delta response, sampling theorem and aliasing are introduced. Noise and stochastic processes are dealt with in Sect. 3.3, which also includes the matched filter. Transversal and recursive filters are discussed in Sect. 3.4. They are followed by Sect. 3.5 on adaptive filters, which includes least mean squares estimation, the Wiener-Hopf filter, the Widrow-Hoff algorithm and lattice filter. The next two sections discuss power spectra estimation and Kalman filtering. Section 3.8 points out the complexity introduced by the two-dimensional systems. The review ends with a discussion of the ambiguity function, the Wigner distribution function and triple correlation.

3.1 Analog Signals and Systems

It is of interest to define the following analog signals at the outset. These functions or signals will be used often in this text and are shown in Fig. 3.1; they are defined as follows:

Step function

\[ u(t) \begin{cases} = 0 & t < 0 \\ = 1 & t \geq 0 \end{cases} \]

Sign function

\[ \text{sgn}(t) \begin{cases} = +1 & t > 0 \\ = -1 & t < 0 \end{cases} \]

Rectangular function

\[ P(t) \begin{cases} = 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ = 0 & t > \frac{1}{2} \end{cases} \]

Delta function

\[ \delta(t) \begin{cases} = 1 & \text{for } t = 0 \\ = 0 & \text{otherwise} \end{cases} \]
Fig. 3.1. Sketches of analog signals (functions)

with
\[ \int_{-\infty}^{+\infty} \delta(t)\, dt = 1 \]

### 3.1.1 Linear Systems

Let us consider a so-called black box with input \( f(t) \) and output \( g(t) \) as shown in Fig. 3.2. The output \( g(t) \) can be represented by a one-to-one mapping function represented by \( L \):
\[
g(t) = L[f(t)]
\]

For a linear system
\[
L[A_1 f_1(t) + A_2 f_2(t)] = A_1 L[f_1(t)] + A_2 L[f_2(t)]
\]
where \( A_1 \) and \( A_2 \) are complex numbers and \( f_1 \) and \( f_2 \) any two functions that satisfy (3.1.1). Also, most of the time we shall deal with time-invariant systems,