Kapitel 7
The Current State of Nonlinear Multiple Criteria Decision Making
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1 Introduction

For many years mathematical optimization models with a single objective function subject to a set of rigid constraints have enjoyed extensive and successful application. These models are relatively simple, and a wide variety of computer software is readily available to efficiently find optimum solutions to such models. However, despite the obvious popularity of this approach, the “optimum” solution it provides may not be the best solution to a real world problem, and in many cases other approaches may be preferable.

In decision making, a model represents a simplification of a real decision situation that is more readily solvable than the original problem. A good model reflects the real situation with sufficient accuracy so that solving the model will provide a good approximation to the solution of the real problem. However, one of the most difficult aspects of constructing good mathematical optimization models is the choice of the objective function. As noted by KLAHR (1958), the difficulty exists not because it is too hard to find an objective function, but rather because it is too easy. He further noted that in many real situations, a number of economic quantities suggest themselves, and to choose only one of them is often both restrictive and arbitrary. In other words, most, if not all, nontrivial real world problems exist within an environment of multiple objectives. These objectives may be incommensurable and conflicting, and subject to both hard (rigid) and soft (flexible) constraints. Multiple criteria decision making or MCDM methodology offers techniques to find solutions to models with multiple objective functions.

Clearly, linear MCDM models provide better approximations to the real world problems than the (single objective) linear programming (LP) formulations. In the same vain, nonlinear MCDM models represent the practical (real) problems more closely than the linear MCDM models. However, linear MCDM problem representations are easier to solve than nonlinear formulations, which partially explains their greater popularity and wider acceptance. In fact, GRIFFITH and STEWART (1961) proposed solving nonlinear MCDM problems via linear approximations. Nevertheless, more recently a number of effective and efficient interactive algorithms have been proposed to solve nonlinear problems directly.

Our objective in this paper is to provide an overview of the current state of the art in nonlinear MCDM methodology and applications. The rest of the paper is organized as follows: In section 2 we present a brief historical perspective. In section 3 we state the various philosophies/approaches used in developing algorithms to solve the problem, followed by comments about several available
algorithms in section 4. In section 5 we report on the current status of software availability, and in section 6 we report on applications. We conclude the paper with a synopsis of our observations and some comments in section 7.

2 Historical Perspective

Nonlinear MCDM methodology has its roots in mathematical programming, the theory of efficient solutions, and the notion of satisficing. Of these, the concept of an efficient solution appears to go back furthest in time. It is also referred to as nondominated solution or Pareto point. It was first used in the context of welfare theory by Pareto (1906) in his manual on political economics and later introduced into other fields by Koopman (1951). The vector maximization problem was first formulated by Kuhn and Tucker (1951), and the idea of satisficing, as opposed to optimizing, was forwarded by March and Simon (1958). Linear goal programming, which in some way represents a satisficing approach, was introduced by Charnes and Cooper (1961). Griffith and Stewart (1961) proposed using linear goal programming for solving nonlinear problems by using repeated linear approximations. Dyer (1972) suggested a generalized form of goal programming with interactive determination of weights to solve the problem.

In its early developmental stages as a discipline, MCDM had to battle for respectability and acceptance. Ignizio (1983) wrote:

... The multiobjective approaches in general were either ignored, overlooked or else dismissed as impractical and the primary focus of attention in the mathematical programming area remained, as it does today, on the traditional and acceptable “single objective” model. During the 1950’s, 1960’s, and even into the mid-1970’s, there were only a relative handful of investigators who seriously pursued and believed in the multiobjective approach. ... 

... Fortunately, a dramatic change in attitude, in regard to the “acceptability of multiobjective mathematical programming” occurred some time toward the mid- to late 1970’s. ... 

Earlier resistance to MCDM methodology might have been due to the fact that, unlike the single objective mathematical program, the multiple objectives model does not possess a mathematically well defined optimum solution. Rather, the decision maker or DM has to choose a compromise solution from among several efficient solutions. Furthermore, compared to the same size single objective problem, a multiple objectives programming problem requires more computational effort. The need for interaction between the DM and the analyst during the solution phase of the problem necessitates the development of interactive and user-friendly procedures and software.

Recent acceptance of MCDM methodology may have been helped by the recognition of the complexity in today’s organizational environment with conflicting interests, incomplete information, and limited resources, in which a DM must attempt to achieve the organization’s objectives (Simon 1960). Furthermore, the growing application of microcomputers in support of business decision making has facilitated the use of interactive algorithms, making it easier to enter