Chapter 8 The Atom in an External Magnetic Field

In this chapter the level splitting in weak and strong magnetic fields (including the splitting of hyperfine structure components) is considered.

8.1 Zeeman Effect

A magnetic field, in contrast to an electric field, completely removes the degeneracy of levels with respect to \( M \). The interaction of an atom with a magnetic field has the form

\[
W = -\mu \cdot H ,
\]

(8.1)

where \( \mu \) is the magnetic moment of the atom. This moment, generally speaking, is composed of two parts — electronic and nuclear. The latter, however, is at least three orders of magnitude less than the former. Therefore, for the magnetic moment of an atom in the state \( \gamma J \) it can be assumed that

\[
\mu = -\mu_0 g J .
\]

(8.2)

Here \( \mu_0 = e\hbar(2mc)^{-1} \) is the Bohr magneton, \( J \) is the total electronic angular momentum, and \( g \) is the gyromagnetic ratio, which is usually called the \( g \) factor (Sect. 6.1). With the \( z \) axis along the direction of \( H \), we obtain

\[
\langle W \rangle = g\mu_0 H M .
\]

(8.3)

Thus the level \( \gamma J \) in a magnetic field splits into \((2J + 1)\) components \( M = 0, \pm 1, \pm 2, \ldots \pm J \). This splitting is linear in \( H \) and is symmetrical. The absolute magnitude of the splitting is determined by the magnitude of \( H \) and the \( g \) factor. As \( g \) is of the order 1, the absolute magnitude of the splitting in cm\(^{-1}\) is \( eH/4\pi mc^2 = 4.7 \cdot 10^{-5} H \). When \( H \) is of the order \( 10^4 \) Oe, the splitting reaches \( 0.5 \) cm\(^{-1}\). The value of the \( g \) factor depends on the type of coupling. In the case of \( LS \) coupling, calculation of the \( g \) factor is very simple. The operator of the magnetic moment of an electron is given by the expression

\[
\mu = -\mu_0 (g_l l + g_s s) ,
\]

(8.4)

where \( g_l = 1, g_s = 2 \); therefore

\[
gJ = \langle g_l \sum_l l_l + g_s \sum_l s_l \rangle = \langle L + 2S \rangle .
\]

(8.5)
Averaging in (8.5) is done over states with a given value of the total angular momentum. Using the equality

\[ L + 2S = J + S \]

and calculating the mean value of \( S \) with the aid of (4.180)

\[ \langle S \rangle = \frac{\langle S \cdot J \rangle}{J(J+1)} J, \] \hspace{1cm} (8.6)

we obtain

\[ g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}. \] \hspace{1cm} (8.7)

This is the so-called Landé factor. When \( S = 0, g = 1 \); when \( L = 0, g = 2 \); and when \( L = S, g = 3/2 \). In the general case for the components of the fine structure of terms with \( L \gg S \)

\[ \frac{L + 2S}{L + S} \gg g \gg \frac{L - 2S + 1}{L - S + 1}, \]

and with \( L < S \)

\[ \frac{L + 2S}{L + S} \leq g \leq \frac{2S + 2 - L}{S - L + 1}. \]

For one electron outside closed shells

\[ g = 1 + \frac{j(j+1) - l(l+1) + 3/4}{2j(j+1)}. \] \hspace{1cm} (8.8)

For some levels (for example, \(*D_{1/2}, *F_{1/2}\) ) the Landé factor is zero. This means that in first-order perturbation theory these levels do not split.

In the case of \( jj \) coupling, calculation of \( g \) factors is a more complex problem. Simple general formulas can be obtained only for the configuration \( jj' \) and \( j^* \). In the first case

\[ gJ = \langle g(j) j + g(j') j' \rangle, \]

\[ g(J) = g(j) \frac{J(J+1) - j'(j' + 1) + j(j + 1)}{2J(J+1)} \]

\[ + g(j') \frac{J(J+1) - j(j + 1) + j'(j' + 1)}{2J(J+1)}, \] \hspace{1cm} (8.9)