of a proximity space proposed by Efremovich (see Engelking (1977)). There exists a canonical one-to-one relationship between compact Hausdorff extensions and proximities. In one direction this relationship can be described in the following way: if \( b(X) \) is a compact Hausdorff extension of \( X \), then the subsets \( A \) and \( B \) of \( X \) are near if and only if their closures in \( b(X) \) intersect (see Engelking (1977)). A similar “connection” exists between the theory of uniform spaces and the theory of compact Hausdorff extensions, however in this case there is no one-to one correspondence. Different uniformities on a Tikhonov space \( X \) can correspond to the same proximity and consequently to the same compact Hausdorff extension (see Arhangel'skii and Ponomarev (1974), Engelking (1977)).

Sklyarenko (see Sklyarenko (1962)) created the theory of perfect compact Hausdorff extensions of Tikhonov spaces. They are characterized by the following property: \( O(U \cup V) = O(U) \cup O(V) \) for any disjoint open subsets \( U \) and \( V \) of \( X \), where \( O(U) \) is the largest open subset of the extension such that \( O(U) \cap X = U \). A compact Hausdorff extension \( b(X) \) is perfect if the Stone-Cech compactification \( \beta(X) \) can be mapped continuously onto \( b(X) \) in such way that inverse images of all points are connected (Sklyarenko, see Sklyarenko (1962)).

Inasaridze (see Inasaridze (1966)) considered the remainders of higher order of Tikhonov spaces. If \( X \) is a Tikhonov space and \( b(X) \) is its compact Hausdorff extension, then \( Y = b(X) \setminus X \) is referred to as the remainder of the first order, \( Z = Y \setminus Y \) the remainder of the second order, \( \overline{Z} \setminus Z \) of the third order, etc. (closures are considered in \( b(X) \)). He clarified, in particular, when a sequence of remainders defined above, terminates. This is associated with the degree to which a space resembles a locally compact space.

§7. Compactness and Spaces of Functions

While the foundations of general topology, and thus also of the theory of compact extensions, belong to the area of set theory and mathematical logic, the credit for the creation and growth of general topology must be given to the theory of functions. Precisely this theory provided (and is still providing) the enlightenment and energy responsible for the development of modern general topology.

One of the sources of general topology, the theory of metric spaces, is a vital part of theory of functions. On the other hand, precisely the ideas of functional analysis are responsible for the growth of general topology beyond the class of metric spaces. These ideas include concepts of pointwise convergence, weak topology, compact-open topology, uniform structure and many others.

It is not surprising, therefore, that many important applications of topological concepts and constructions belong to the area of functional analysis and that several fundamental principles, associated with general topology, have
a twofold, topologically-functional or linear-topological, character. Such results include the theorems of Stone-Weierstrass, Krein-Milman, Arzela-Ascoli, Alaoglu, Markov-Kakutani and many others.

In this section we provide a review of such principles and discuss their role in general topology and its applications.

7.1. Natural Topologies on Spaces of Functions. We will denote by $\mathbb{R}^X$ and $C(X)$, respectively, the sets of all real-valued and the set of all continuous real-valued functions on a topological space $X$. There are several different natural ways of introducing a topology on $\mathbb{R}^X$ and $C(X)$ and they all follow the general approach indicated below.

Let $S$ be a family of subsets of $X$. We assign to each finite sequence $P_1, \ldots, P_k$ of elements of $S$ and to each sequence (of the same length) $U_1, \ldots, U_k$ of open subsets of $\mathbb{R}$ the set $O'(P_1, \ldots, P_k; U_1, \ldots, U_k)$ consisting of all $f \in \mathbb{R}^X$ for which $f(P_i) \subset U_i$, for all $i = 1, \ldots, k$. Put $O(P_1, \ldots, P_k; U_1, \ldots, U_k) = C(X) \cap O'(P_1, \ldots, P_k; U_1, \ldots, U_k)$. Sets of this form are a base of some topology on $\mathbb{R}^X$, respectively on $C(X)$, called the topology of convergence with respect to elements of $S$. Under this construction the space $C(X)$ is a subspace of the space $\mathbb{R}^X$. The family $S$ is typically assumed to be quite rich, for example, one can require that it be a network of $X$. This condition assures sufficiently good separation properties of $\mathbb{R}^X$ and $C(X)$. The most important topologies obtained in this way are the following: the topology of pointwise convergence corresponding to the case when $S$ consists of all one-point (or all finite) subsets of $X$, and the compact-open topology arising when $S$ consists of all compact subsets of the space $X$. If $S$ consists of all bounded subsets of $X$ (recall, that those are the sets $A \subset X$ on which every continuous, real-valued function on $X$ is bounded) then the topology of convergence with respect to $S$ is called the bounded-open topology. The following result is of fundamental importance.

**Proposition 1** (see Bourbaki (1969a)). *For every Tikhonov space $X$ the compact-open topology, the topology of pointwise convergence and the bounded-open topology are compatible with the natural linear structure on the space $C(X)$ and they induce on it the structure of a locally convex linear topological space over $\mathbb{R}$.***

We will denote by $C_k(X), C_0(X)$ and $C_p(X)$, respectively, the set $C(X)$ equipped with the compact-open topology $T_k$, bounded-open topology $T_0$ and the topology of pointwise convergence $T_p$.

The topologies of uniform convergence on $\mathbb{R}^X$ and $C(X)$ are described in a slightly different way. Their bases consist of the sets $O'_\varepsilon(f) = \{g \in \mathbb{R}^X : \sup_{x \in X} |f(x) - g(x)| < \varepsilon\}$ and $O_\varepsilon(f) = O'_\varepsilon(f) \cap C(X)$, respectively. However, if a Tikhonov space $X$ is not pseudocompact, then the topology of uniform convergence is not compatible with linear operations on $C(X)$. Indeed, if $f$ is an unbounded, continuous function on $X$, then $0 \cdot f = \theta$ is a function that is