AN OPTIMUM CONCEPT FOR FUZZIFIED LINEAR PROGRAMMING PROBLEMS*

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Abstract: In this paper an optimality concept is introduced for the (g,p,d)-fuzzified linear programming problems. It is based on the parametrical embedding method of the classical nonlinear programming.

Zusammenfassung: In dieser Arbeit stellen wir ein Optimalitätskonzept für (g,d,p)-fuzzifizierte Probleme der linearen Programmierung vor, das auf der parametrischen Einbettungsmethode der klassischen nichtlinearen Programmierung beruht.

1. Introduction

Let us consider the classical linear programming problem

$$\sum_{j=1}^{n} \gamma_j x_j \rightarrow \min$$

subject to

$$C = \{ x \in \mathbb{R}^n : \sum_{j=1}^{n} a_{ij} x_j = \alpha_i 0, \quad i=1,\ldots,m, \}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq \alpha_i 0, \quad i=m+1,\ldots,s \}. \quad (2)$$

In the problem (1)-(2) the parameters $\gamma_j, a_{ij} \in \mathbb{R}$ are supposed to be well defined characteristics of the modelled problem. However, these parameters are generally known only approximatively.

In this paper instead of (1)-(2) we will examine the fuzzified version of (1)-(2) assuming that the parameters in the problem formulation are given by fuzzy numbers. The support set of a fuzzy number describes the set of possible values of the perturbed parameters. The fuzzification concept is based on the idea that the uncertainty in the real world has a functional causality and this functional causality takes effect in the choice of parameters as well as in the arithmetic operations. The optimum concept is motivated by the parametrical embedding method for the classical nonlinear programming /3/.

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2. Basic definitions and theorems

Let \( X \) be a universe and \( I \) be the unit interval of the real line \( \mathbb{R} \). Denote \( \mathcal{G}(X) = \{ \mu | \mu : X \to I \} \) the set of all fuzzy subsets on \( X \).

The characteristic function of \( A \subseteq X \) as a special fuzzy subset will be denoted by \( \chi_A \).

A binary operation \( T : I^2 \to I \) is said to be a t-norm iff it is commutative, associative, non-decreasing and \( T(a,1) = a \) for all \( a \in I \). A t-norm is Archimedean iff it is continuous and \( T(a,a) < a \) for all \( 0 < a < 1 \).

A t-norm \( T \) is Archimedean iff it admits the representation
\[
T(a,b) = g(-1)(g(a)+g(b))
\]
where the generator function \( g : I \to \mathbb{R}_+ \) is continuous, strictly decreasing, \( g(1) = 0 \) and \( g'(x) \) denotes the pseudoinverse of \( g \), i.e. \( g'(x) = \frac{1}{g(x)} \) for all \( x \in [0, g(0)] \) and \( g'(x) = 0 \) for all \( x \not\in [0, g(0)] \). Every t-norm induces an \( n \)-ary operation \( T^{n-1} : I^n \to I \) with the following rule
\[
T^{n-1}(a_1, a_2, \ldots, a_n) = T(T^{n-2}(a_1, a_2, \ldots, a_{n-1}), a_n).
\]
If \( T \) is Archimedean then
\[
T^{n-1}(a_1, a_2, \ldots, a_n) = g(-1)(\sum_{j=1}^{n} g(a_j)).
\]

The \( T \)-intersection and \( T \)-Cartesian product of fuzzy sets will be defined as follows: if \( \mu, \nu \in \mathcal{G}(X) \) then \( (\mu \land \nu)(x) = T(\mu(x), \nu(x)) \), if \( \mu \in \mathcal{G}(X) \), \( \nu \in \mathcal{G}(Y) \) then \( (\mu \times \nu)(x,y) = T(\mu(x), \nu(y)) \).

Let \( g : I \to \mathbb{R}_+ \) be a fixed function with the properties of a generator function and let \( \mathcal{G}_g \) denote the subset of \( \mathcal{G}(\mathbb{R}) \) containing the fuzzy sets with the membership function
\[
\mu(a) = \begin{cases} 
    g(-1)(|a - \alpha|/d) & \text{if } d > 0 \\
    \chi_{[\alpha]}(x) & \text{if } d = 0
\end{cases}
\]
for all \( \alpha \in \mathbb{R}, \ d \in \mathbb{R}_+ \cup \{0\} \). The elements of \( \mathcal{G}_g \) will be called quasi-triangular fuzzy numbers generated by \( g \) with the center \( \alpha \) and width \( d \) and we will refer to them by pairs \((\alpha, d)\). Let \( \mathcal{G}_g \subseteq \mathcal{G}_g \) be the subset of all \( d \) width quasitriangular fuzzy numbers generated by \( g \).

Let \( T_{g,p} \) be an Archimedean t-norm given by the generator function \( g^p \); \( 1 \le p < \infty \). It is easy to see that \( \lim_{p \to \infty} T_{g,p}(a,b) = \min(a,b) \), therefore we will also use the notation \( T_{g,p} \) in the case \( p = \infty \) meaning the min-norm for \( T_{g,\infty} \).

The \( T_{g,p} \)-Cartesian product of \( n \) quasitriangular fuzzy numbers generated by \( g \) will be called \((g,p,d)\)-fuzzy vector. Let \( \mu = \mu_1 \times \ldots \times \mu_n \) be a \((g,p,d)\)-fuzzy vector, where \( \mu_j(\alpha, d) \in \mathcal{G}_g \), \( j = 1, \ldots, n \). It is obvious