On the General Theory of Quantized Fields

dedicated to the memory of Res Jost (†1990)

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1 The Aim of the General Theory of Quantized Fields

The term “General Theory of Quantized Fields”, replacing the synonymous but somewhat misleading term “Axiomatic Field Theory”, is to my knowledge due to Res Jost. He was one of the great pioneers in our field, and I dedicate this lecture to his memory. What is the aim of the general theory of quantized fields?

An answer may be found in Jost’s famous book with the same title [1]. Jost formulates it as follows: “The general theory of quantized fields analyzes the notions which are at the basis of all previously analyzed specific models”. So the general theory of quantized fields is concerned with model independent aspects of quantum field theory. Actually, the richness of the general theory of quantized fields is intimately connected with the notorious difficulties in constructing and evaluating examples of quantum field theories.

There has been a lot of progress in quantum field theory since the time the project of the general theory of quantized fields was started by the work of Wightman, Haag, Lehmann, Symanzik, Zimmermann and others. Non-abelian gauge theories became the most promising models for elementary particle physics, renormalization has much better been understood by now,
interacting models in low dimensions were constructed, numerical approximation on lattices and semiclassical arguments supplement the perturbation theoretical approach, and experiments in elementary particle physics have supported much of the theoretical predictions. More recently, a rich class of interesting models was detected in 2 dimensional conformal field theory, integrable models with exact S-matrices were found, and string theory and topological field theory broadened the range of quantum field theory.

In spite of all these successes, quantum field theory remains a very difficult subject, and the general analysis of structural properties is often the most convenient way to derive predictions in combination with some input from the models or even directly from experiment.

In my lecture I want to describe the present stage of the general theory of quantized fields on the example of 5 subjects. They are ordered in the direction from large to small distances.

The first one is the by now classical problem of the structure of superselection sectors. It involves the behavior of the theory at spacelike infinity and is directly connected with particle statistics and internal symmetries. It has become popular in recent years by the discovery of a lot of nontrivial models in 2d conformal field theory, by connections to integrable models and critical behavior in statistical mechanics and by the relations to the Jones' theory of subfactors in von Neumann algebras and to the corresponding geometrical objects (braids, knots, 3d manifolds,...).

At large timelike distances the by far most important feature of quantum field theory is the particle structure. This will be the second subject of my lecture. It follows the technically most involved part which is concerned with the behavior at finite distances. Two aspects, nuclearity which emphasizes the finite density of states in phase space, and the modular structure which relies on the infinite number of degrees of freedom present even locally, and their mutual relations will be treated.

The next point, involving the structure at infinitesimal distances, is the connection between the Haag-Kastler framework of algebras of local and the framework of Wightman fields. Finally, problems in approaches to quantum gravity will be discussed, as far as they are accessible by the methods of the general theory of quantized fields.

2 Sectors, Statistics and Symmetry

The theory of superselection sectors has ever been a challenge since the observation of Wick, Wightman and Wigner [2] that the superposition principle in quantum physics does not hold unrestrictedly. It was formulated as a classification problem on the representations of the algebra of observables by Haag and Kastler [3], was initiated by Borchers [4] and reached a certain degree of completeness in the work of Doplicher, Haag and Roberts [5].