3. Limitations of Fermi Theory

3.1 Neutral Currents

We noticed that there are no scattering processes of the form $\nu_\mu e^- \rightarrow \nu_\mu e^-$ or $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ in the framework of Fermi’s theory with V–A coupling. One therefore has to carefully investigate experimentally whether such scattering occurs in nature. These experiments are extremely difficult, because the expected cross sections (if any) lie in the range $10^{-41} - 10^{-44}$ cm$^2$ ($10^{-17} - 10^{-20}$ barn). Only with the high neutrino currents in modern accelerators (Fermilab near Chicago, CERN - SPS) and high neutrino energies (several hundred GeV) did such experiments become practical at all.

In fact many such processes were observed; the best experimental values for the cross sections are:

$$\frac{1}{E_{\nu}} \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) = (1.45 \pm 0.26) \times 10^{-42} \text{ cm}^2/\text{GeV} \quad (3.1a)$$

$$\frac{1}{E_{\bar{\nu}}} \sigma(\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-) = (1.3 \pm 1.0) \times 10^{-42} \text{ cm}^2/\text{GeV} \quad (3.1b)$$

The existence of such so-called “neutral” currents can therefore be regarded as being firmly established. Here the name “neutral current” has the following origin. If one starts from the conservation of electron and muon number separately, the only possible interpretation of the scattering process $\nu_\mu e^- \rightarrow \nu_\mu e^-$ is that at the interaction point the incoming electron turns into the outgoing electron and the incoming $\nu_\mu$ neutrino turns into the outgoing $\nu_\mu$ neutrino. The obvious method to implement this process in our theory is therefore to supplement the leptonic current $J^{(L)}_{\mu}$ by expressions of the form

$$\bar{u}_\nu \gamma_\alpha (1 - \gamma_5) u_\nu \quad (3.2a)$$

$$\bar{u}_e \gamma_\alpha (g_\nu - g_A \gamma_5) u_e \quad (3.2b)$$

Here we have made use of the fact that in any event neutrinos must have negative helicity. The current (3.2a) does not contain a charged particle at all, that is it is really “neutral”, while in (3.2b) the charge of the particle is conserved, which one also somewhat sloppily refers to as “neutral” (in this sense the electromagnetic current $\bar{u} \gamma^\alpha u$ is “neutral” for all particles!). Important is the fact that the incoming particle changes its charge in the charged transition currents as occurs in (2.1) and (2.4). This is not the case with neutral currents.

Fig. 3.1. Neutrino-electron scattering is not possible in the context of V–A theory, as developed so far.
3.2 Scattering of a Muon Neutrino by an Electron

We now calculate the cross section for \( \nu_\mu e^- \rightarrow \nu_\mu e^- \), starting from the currents (3.2). The relevant interaction term is:

\[
H_{\text{int}}(\nu_\mu e^- \rightarrow \nu_\mu e^-) = \frac{G}{\sqrt{2}} \int d^3 x [\bar{u}_{\nu_\mu} \gamma^\alpha (1 - \gamma_5) u_{\nu_\mu}] [\bar{u}_e \gamma_\alpha (g_V - g_A \gamma_5) u_e] .
\]  

Because the cross section is incredibly small, it is impossible to observe the helicity of the electrons before or after the scattering process. Hence it is sufficient to calculate the averaged cross section, where one averages or sums over all helicities. In addition, it is convenient to integrate over all momenta of the outgoing particles, since these are also not measurable in practice (the momentum \( p' \) of the electron could in principle be measured, but the statistics of such a differential experiment would be completely insufficient).

We then obtain the following result (the details of the calculation are the subject of Exercise 3.1):

\[
\bar{\sigma} = \frac{G^2}{8 \pi^2} \frac{1}{16(k \cdot p)} \int \frac{d^3 k'}{k_0'} \int \frac{d^3 p'}{p_0'} \delta^4(p' + k' - p - k) \frac{1}{2} \sum_{s, s'} |M|^2 ,
\]  

where

\[
M = [\bar{u}_{\nu_\mu}(k', t') \gamma^\alpha (1 - \gamma_5) u_{\nu_\mu}(k, t)] [\bar{u}_e(p', s') \gamma_\alpha (g_V - g_A \gamma_5) u_e(p, s)] .
\]

The individual parts of the matrix elements are now evaluated exactly as in the case of muon decay (cf. Chap. 2). We start with that of the neutrino:

\[
\sum_{t, t'} \bar{u}_{\nu_\mu}(k', t') \gamma^\alpha (1 - \gamma_5) u_{\nu_\mu}(k, t) \bar{u}_{\nu_\mu}(k, t) \gamma_\beta (1 - \gamma_5) u_{\nu_\mu}(k', t')
\]

\[
= \text{Tr} \{ \gamma^\alpha (1 - \gamma_5) \bar{\gamma}^\beta (1 - \gamma_5) \} = 2 \text{Tr} \{ \gamma^\alpha \bar{\gamma}^\beta (1 - \gamma_5) \}
\]

\[
= 2k_\mu k'_\mu \text{Tr} \{ \gamma^\alpha \gamma^\mu \bar{\gamma}^\beta \gamma_\beta (1 + \gamma_5) \}
\]

\[
= 8(k^\alpha k'^\alpha - g^\alpha \beta (k \cdot k') + k^\beta k'^\alpha + i e^\alpha \beta \gamma^\nu k_\mu k'_\nu) .
\]

Analogously we find, that for the electronic part

\[
\sum_{s, s'} \bar{u}_e(p', s') \gamma_\alpha (g_V - g_A \gamma_5) u_e(p, s) \bar{u}_e(p, s) \gamma_\beta (g_V - g_A \gamma_5) u_e(p', s')
\]

\[
= \text{Tr} \{ \gamma_\alpha (g_V - g_A \gamma_5) \bar{\gamma}_\beta (g_V - g_A \gamma_5) + m_e \gamma_\beta \}
\]

\[
= \text{Tr} \{ \gamma_\alpha \bar{\gamma}_\beta \gamma_\beta \gamma (g_V + g_A \gamma_5)^2 + \gamma_\alpha \gamma_\beta m_e (g_V + g_A \gamma_5) (g_V - g_A \gamma_5) \}
\]

\[
= \text{Tr} \{ \gamma_\alpha \bar{\gamma}_\beta \gamma_\beta \gamma (g_V^2 + g_A^2 + 2 g_A \gamma_5 \gamma_5) + m_e^2 \text{Tr} \{ \gamma_\alpha \gamma_\beta \} \}
\]

\[
= 4 \left[ (g_V^2 + g_A^2) (p_\alpha p_\beta + p_\beta p_\alpha) + 2 i g_A \gamma_5 \varepsilon_{\alpha \beta \gamma \delta} p_\gamma p_\delta \right] + 2 i g_A \gamma_5 \varepsilon_{\alpha \beta \gamma \delta} p_\gamma p_\delta
\]

\[
= m_e^2 (g_V^2 - g_A^2) g_\alpha g_\beta ,
\]

where we have made use of the fact that the trace of an odd number of \( \gamma \) matrices vanishes.