8 Non-linear Decay Interactions

8.1 General Concepts

The simplest decay process is a wave-wave emission process, that is, a non-linear interaction in which a wave of a particular type decays and produces another wave of the same type as well as a wave of a different type. An example is the decay of a Langmuir wave into another Langmuir wave and an ion-sound wave. This process can be considered to be the emission of an ion-sound wave by a Langmuir wave. Symbolically we can write this process in the form

\[ \ell \rightarrow \ell' + s. \]  
(8.1)

This process is shown in Fig.8.1.

Let the initial momentum of the Langmuir wave be \( k' \), the momentum of the emitted ion-sound wave be \( k \), so that the final momentum of the Langmuir wave is \( k' - k \). The energy conservation law for this wave-wave emission process is:

\[ \omega_{k'} = \omega_{k'-k} + \omega_k. \]  
(8.2)

This is similar to the equation when a wave is emitted by a particle; in particular, the emission of ion-sound waves by particles is described by the equation

Fig. 8.1. A wave-wave emission process
\[ \varepsilon_p = \varepsilon_{p-k} + \omega^q_k. \]  

(8.3)

In both cases we have for the case when the momentum of the emitted wave is small as compared to the initial momentum of the incoming source either the condition for Vavilov-Cherenkov emission, 

\[ \omega^q_k = (k \cdot v), \]  

(8.4)

or the condition for group wave resonance, 

\[ \omega^q_k = (k \cdot v_{gr}), \]  

(8.5)

where here \( v_{gr} \) is the group velocity of the Langmuir waves.

\[ v_{gr} = \frac{d \omega^q_k}{dk}. \]  

(8.6)

One might say that in some sense Eq.(8.5) is the Cherenkov condition for wave-wave emission, showing that if the process is the one where an ion-sound wave is emitted by Langmuir waves the group velocity of the Langmuir wave responsible for the emission should be larger than the phase velocity of the ion-sound waves. If we proceed as in the case of the emission of waves by particles where we introduced a probability \( w_p(k) \) and now introduce a probability \( w_{k'}(k) \) for the emission of ion-sound waves, we can write down an equation similar to the quasi-linear equation which takes into account both the spontaneous and the stimulated processes. In some approximations this equation differs from the quasi-linear equation in the notation used and, of course, in the probability which occurs in it. It looks as if the whole problem consists in finding an expression for the probability for the emission of waves by waves. However, the problem is not as simple as that. In the case of the emission of waves by particles the momentum of the wave is practically always smaller than that of the particle, but this is not so in the case of the emission of waves by waves when the momentum of the emitted wave can be comparable with the momentum of the source wave. This is, for instance, the case for the emission of ion-sound waves by Langmuir waves. The probability for the emission of an ion-sound wave is the largest if in the final state the Langmuir wave propagates in a direction opposite to that of its initial direction with the same absolute magnitude of its momentum. The emitted ion-sound wave then has a momentum equal to twice that of the initial momentum of the Langmuir wave. There are also differences in the statistical properties of the source of the emission between the emission of waves by waves and the emission of waves by particles.

Among all possible decay processes we have not only the emission of waves, but also more complicated processes, for instance, the decay of a single wave into more than two waves. Another possibility is that two waves are changed into two other waves, and so on. Examples of those processes are the processes: