Dissipative Environments: Quantum Jumps or Phase-Space Localization?

B.M. Garraway and P.L. Knight

Optics Section, The Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

Abstract. We discuss the evolution of individual realizations of quantum superpositions conditioned by two distinct measurement schemes: amplitude (heterodyne) measurements and excitation number (counting) measurements. These two schemes result in either localization or quantum jumps in phase space, which we illustrate for some non-classical initial superpositions.

1 Introduction

The decay of coherence when a quantum system interacts with a much larger environment is usually described by a master equation for the system reduced density matrix, and emphasises the evolution of an entire ensemble [1]. Coherences are destroyed at a rate which is proportional to the degree of excitation of the system (much faster than energy decay) as the system relaxes to a statistical mixture of relevant "pointer bases" [2,3]. Such an ensemble-averaged approach does not easily address the question of how an individual member of an ensemble evolves in a dissipative environment, even though this issue is accessible to experiment [4]. A number of methods have been developed recently to simulate the evolution of single realizations. One centres on quantum jumps, reflecting the information gained from the detection of decay quanta, and involves a continuous evolution which is randomly interrupted by "instantaneous" jumps as the system state vector is conditioned by the information gained from the register of counts in detection of decay [5,6]. The other method involves both a quantum state diffusion where the individual quantum trajectory fluctuates through a Wiener process deriving from the environment, and localization to a coherent state, an eigenstate of the relevant Lindblad operator describing the coupling of the system to the environment [7]. Wiseman and Milburn [8] have related these two simulation methods to specific measurement schemes: the quantum jump method to continuous direct counting of decay quanta, and the quantum state diffusion method to a heterodyne measurement of oscillation amplitudes. Both methods, of course, lead to the same ensemble behaviour when averaged over many realizations [5–9]. The new simulation schemes have been widely adopted for their utility in computing averages in a numerically efficient way on small computers. A quite separate issue is what, if any, meaning can be ascribed to individual trajectories, for these do differ in
the different simulation schemes, reflecting the distinctly different conditionings applied, and reflecting the acquisition of different information in each simulation. Individual quantum trajectories “unravelled” from the master equation reflect the particular measurement scheme [6,8].

Here, we examine single realizations of a superposition of coherent states of an oscillator (for example, a single electromagnetic field mode) interacting with a dissipative environment. We show that such realizations evolve very differently in the two simulation methods described above. The quantum jump, or state vector Monte-Carlo, method can lead to what we have called [10] a “jumping cat” where the conditioned state vector evolves through a series of jumps from one pure state “cat” (e.g. an even coherent state) to another (an odd coherent state) as individual quanta are removed at random by the dissipation. The quantum state diffusion model leads to a rapid localization to one or other of the coherent state components of the initial superposition, reflecting the approximate projection onto one potential outcome of an amplitude measurement. The coherent states are an overcomplete set and form an interesting pointer basis for an amplitude (heterodyne) measurement on any state: we exploit this in a study of individual trajectories of a chosen state (a Fock state) which localizes to one coherent state component (picked at random) of that state expressed in a coherent state basis. We employ phase-space descriptions of the evolution, and conclude with a comment on the “tiling” of phase-space and localization.

In order to compare the simulation methods with the density matrix result we need to solve the master equation for that density matrix. In a typical dissipative process the master equation for \( \rho \) takes the form (with \( \hbar = 1 \))

\[
\frac{d}{dt} \rho = -i [H, \rho] + \left( \hat{R} \rho \hat{R}^\dagger - \frac{1}{2} \hat{R}^\dagger \hat{R} \rho - \frac{1}{2} \rho \hat{R}^\dagger \hat{R} \right),
\]

where the Lindblad reservoir operator \( \hat{R} \) represents the influence of the environment. We have discussed elsewhere [9] the case of a two-photon absorber but \( \hat{R} \) would typically be \( \sqrt{\kappa} \hat{a} \) for ordinary single photon dissipation. In the case of a dissipative two-level system we would have \( \hat{R} = \sqrt{\kappa} \sigma_- \) where \( \sigma_- \) is one of the Pauli spin matrices. In both of these cases the parameter \( \kappa \) signifies the loss rate of energy into the reservoir and the Hamiltonian \( H \) would represent the coherent evolution of the lossless system. The methods used to solve (1) depend specifically on the choice of operators \( \hat{R} \) and the Hamiltonian \( H \). Sometimes there is more than one reservoir operator, e.g. for a cavity in a thermal heatbath (characterised by a photon occupation \( \bar{n} \)) we have both \( \sqrt{\bar{n} + 1} \kappa \hat{a} \) and \( \sqrt{\bar{n}} \kappa \hat{a}^\dagger \) as reservoir operators. However, all the methods we describe below can be extended to cope with this kind of situation.

### 2 Quantum Jumps

When we perform a simulation of a density matrix problem (by quantum jumps or quantum state diffusion) we consider not the density matrix \( \rho \), but a state