Relationship Between Perturbation Size and Structure of the Vortex Pair for Converging Cylindrical Shocks

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Abstract. Converging cylindrical shock front shapes, bearing small and large perturbations, were examined via spark Schlieren photography taken near the geometric center. The perturbation parameter was found to increase with shock travel, according to $r^{-n}$ but with fluctuations that are found to be linearly proportional to its initial value. Based on the hypothesis that the distance between the vortex pairs is proportional to the radius where breakdown in shock front curvatures takes place, a relationship was obtained between the distance separating the two vortex pairs and the initial value of the perturbation. This relationship was found to be in excellent agreement with those experimentally measured. Both theory and experiments were also found to approach a constant value as the perturbation on the shock front increases beyond a certain limit.

Key Words: Converging cylindrical shocks, Stability of converging shocks, Vortex growth

1. Introduction

Recent work on converging cylindrical shocks indicates that they are inherently unstable. Weak converging ones were successfully produced by the tear-drop apparatus of Perry and Kantrowitz (1951) while stronger ones, possessing a high degree of symmetry, were only produced employing other designs (Wu et al. 1981, Takayama et al. 1987). None, however, maintained their circular shape until collapse. Breakdown in their front curvature was evident in all cases due to the subsequent formation of vortices behind the expanding shock.

Lately, a great number of research work has been oriented towards the stability of converging shocks. According to Butler (1956), all converging shocks are unstable. The stability parameter $\xi$, defined as $\Delta R/R_s$, was found to increase monotonically according to $r^{-n}$. A similar solution was later obtained but was illustrated in an oscillatory manner that seemed unrealistic at the time due to the lack of experimental evidence (Gardner et al. 1982).

Experimentally, the stability of converging shocks was studied by perturbing them externally through the use of cylindrical rods placed in their path before collapse. Theoretically, the shock front shape and the triple point trajectories were successfully determined from wave diagrams constructed using the Ray-Shock theory (Neemeh and Le 1989). The variation in $\xi$, calculated from these diagrams, was found to fluctuate in a harmonic fashion before finally increasing to large values before collapse. The present work was therefore carried out to determine a relationship which better describes the stability of converging cylindrical shocks for a wide range of initial perturbations. When the shock is highly symmetrical an alternate method was employed, due to the difficulty in measuring the degree of instability from the Schlieren photographs. The distance separating the two pairs of vortices, trailing behind the expanding shock, was used to determine the latter. The details are presented in the following section.

2. Experiments

The apparatus used in the present work is a 6-inch cylindrical shock tube which employs a three-increment area contraction to turn, while amplifying, annular plane shocks 90 degrees to their original direction of propagation (Wu et al. 1981). In the present work, the converging shocks were perturbed by narrowing the cylindrical chamber's width which in turn amplifies the effects...
of the asymmetries that could be present in the apparatus. Schlieren photos of the flow were produced using a double-headed spark Schlieren system, equipped with a delay circuit. Four sets of photos are presented in Fig.1 for converging shocks with different initial perturbation, $\xi_0$, of 0.006, 0.031, 0.065 and 0.08, measured at radius ($R_0$) of 11.3 mm. These sets were produced from an initially plane shock Mach number of 1.24.

From these photographs and many others, not shown in the figure, it was possible to measure the shock front perturbation parameter $\xi$. The results are presented in Figs.2, 3, 4 and 5.

From these results, it is clear that $\xi$ cannot be represented by the simple $r^{-n}$ relationship. A harmonic type of variation is clearly noted specifically in the larger perturbation case (Fig.5). These variations, however, are not as large as those theoretically predicted by Gardner et al. (1982) but are, nevertheless, present to a degree which depends on the size of the initial perturbation. By taking the Butler solution (Butler 1951) and assuming a harmonic component that varies according to the relation: