THE ENGINE SCHEDULING PROBLEM IN A RAILWAY NETWORK
PART I

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Summary

In this paper we present the mathematical version of a scheduling problem that is faced by any railway company that employs several engine types to provide power for its trains. Usually, a railway employs engines of several types that differ in their tractive effort capability and horsepower rating. There are two distinct but related aims in reducing the engine costs to the railway. One is to select the mix of engine types that gives the lowest capital investment and operating cost for the trains operated by the railway. The other is to provide a scheduling method that assigns available engines to trains on a short time horizon (e.g. a week ahead). We describe the practical context of the former problem below.

A railway runs many of its trains to a predetermined schedule that is publicly available to buyers of their service. The convenient graphical representation of the scheduled trains is the widely used time space diagram. This diagram is a network that is plotted to scale using two coordinates: the horizontal coordinate is time and the vertical coordinate is
Thus, a train is specified on the abscissa by its departure time and arrival time and on the ordinate, by the originating and terminating cities. Train arcs are diagonal arcs in the network, when this convention is used. Horizontal arcs are added to the network to connect the terminal node of a diagonal arc to initial nodes of other diagonal (train) arcs that originate at the same city (station).

The flows associated with each diagonal arc are the engines that are assigned to the corresponding train, and with horizontal arcs engines that are idle and waiting to be reassigned to another train that originates at the same city. Empty or "dummy" trains may also be indicated on the time space diagram. Figure 1 gives an example of a time space diagram for 3 cities and 4 trains. The "smaller" arcs such as (2,3) and (4,5) represent the movement of engines within the railway terminal, starting at the time they arrive in a yard and ending at the time they are ready to be reassigned. Similarly, arc (11,12) represents the flow of engines that stays with the continuation of train 3 from city B to city A.

Each train \((x,y)\) has a motive power requirement \(T(x,y)\) that is determined by the weight and length of the train. Every engine type \(i\), for train \((x,y)\) has a tractive effort rating \(s_i(x,y)\) that is related to engine design and steepness of grades over which the train must be pulled. In addition, every engine type \(i\),