11. Hot-Electron Transport

11.1 Phenomenon of Hot Electrons

Electron transport is caused by an electric field, a thermal gradient, or a concentration gradient. In the latter two cases, a uniform condition is established as a result of the transport unless external sources are used to maintain the nonuniform conditions. The transport of electrons under these conditions may cause transfer of energy from one part of the sample to the other, but the electrons do not gain energy from the external sources in the process of transport. On the other hand, when the transport is caused by an electric field, electrons are continuously supplied with energy from the source of the electric field at a rate \( \mathbf{J} \cdot \mathbf{E} \) (\( \mathbf{J} \) is the current density and \( \mathbf{E} \) is the electric field), and it would appear that the total energy of the electron system should go on increasing indefinitely. This, however, does not happen as this gain of energy is balanced by transfer of energy to the lattice atoms through the process of collisions. It has been seen that an electron is scattered by the lattice either by emitting or by absorbing a phonon. The lattice absorbs energy from the electron when a phonon is emitted and it delivers energy to the electron when a phonon is absorbed. In the absence of the electric field the absorption and emission processes are so balanced that there is no net transfer of energy from the electron system to the lattice system or the vice versa. It means in effect that the temperature, the thermodynamic coefficient determining transfer of energy from one system to another, of the electron and that of the lattice system are identical. One may verify easily from the Boltzmann equation that in the field-free condition,

\[
\int \mathcal{E} \left[ S(k, k')_{ab} f_0(k) (1-f_0(k')) + S(k', k)_{ab} f_0(k') (1-f_0(k)) \right] dk' dk
= \int \mathcal{E} \left[ S(k, k')_{em} f_0(k) (1-f_0(k')) + S(k', k)_{em} f_0(k') (1-f_0(k)) \right] dk' dk . \tag{11.1}
\]

for the same value of temperature in the electron distribution function and in the phonon occupation number. On the other hand, in the presence of an external electric field, the energy of the electron system starts to increase.
immediately after the application of the field. But as the energy increases, electrons emit more phonons than they absorb, and there is a net transfer of energy from the electron system to the lattice system. Referring back again to the Boltzmann equation, we find that if the temperature $T_e$ of the electron system is larger than the lattice temperature $T_L$,

$$\int E(\partial f/\partial t)C_d k > 0 \quad ,$$

and this transfer continues to increase as the difference between $T_e$ and $T_L$ increases with time. A new equilibrium may therefore be expected instead of a continuous increase of the energy of the electrons. This equilibrium is established when the difference between the electron and the lattice temperature is such that the rate of gain of energy of the electrons from the electric source ($\mathbf{J} \cdot \mathbf{E}$) is balanced by the rate of loss of energy to the lattice atoms, i.e. when

$$\mathbf{J} \cdot \mathbf{E} = (4\pi^3)^{-1} \int E(\partial f/\partial t)C_d k$$

We thus find that when electron transport occurs under the action of an electric field, the electron temperature is required to be higher than the lattice temperature to produce a steady state condition. The statement should remain valid even when the electric field is low. But, for low fields the required rise in electron temperature is so small that the difference between the electron and the lattice temperature does not produce any measurable change in the values of the transport coefficients. We may hence neglect this rise for low electric fields and take the electron temperature to be identical to the lattice temperature even when the electric field is applied. The theory of electron transport has been developed in the earlier chapters on this assumption. A consequence of this was that the symmetric part of the electron distribution function could be taken to be identical to the equilibrium function and the current density was found to vary linearly with the electric field. In other words, it is due to the validity of this assumption that we find the current-voltage relation to be given by Ohm's law at low fields.

As the electric field is increased, the difference between the electron and the lattice temperature becomes significant. The current density starts to vary nonlinearly with the electric field as it increases, even if the lattice temperature and the electron concentration are kept unaltered. The nonlinearity may be approximated by a square law,