About the Periodic Contact Problem for the Maxwell Body

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Summary
This paper is devoted to the periodic problem for a Maxwell body submitted to unilateral constraints. A similar subject was already treated by author's group ([1],[2],[3]) about periodic stresses in cracked turbine blades. The unilateral constraint was the non-negativity of the crack opening displacement.

The new results brought by this paper are the complete formulations of the periodic problem for any convex domain of unilateral constraints as well as a new technique which allows to reach the solution directly, that is to say with no regularization of the constitutive equation. Due to this technique that exactly fits the liquid character of the Maxwell body, the shape of the obstacle can be as irregular as wanted. It is hoped that the tools used here for the case of a beam can be generalized to other cases.

§ 1. Naive equations
Consider a straight beam, with unity length, flexured in the (x, O, y) plane as represented by the opposite figure. According to the simplest flexion theory the deflection \( u(x) \) is small and parallel to the y-axis. The generalized strain \( e \) is the cur-
vature $\frac{\partial^2 u}{\partial x^2}$, associated with the generalized stress $s$ which is the bending moment.

This beam is fixed at both ends and submitted to a given imposed strain $-\varepsilon_0$, and to a given lineic density of forces $f^0$; both of them depend on $(x, t)$. Moreover a fixed obstacle is given by its equation $y = a(x)$ and the beam must remain above it. A suitable generality is obtained by allowing the function $a$ to take the value $-\infty$.

The quasi-static evolution of such a beam is ruled by the following system:

$$
\begin{align*}
\frac{\partial^2 u}{\partial x^2} &= -\varepsilon_0, \\
\frac{\partial^2 s}{\partial x^2} &= f^0 + g
\end{align*}
$$

(3)

The last equation describes the behaviour of the material: it is a functional relation with respect to $t$, pointwise with respect to $x$. To shorten the exposition let us restrict ourselves to the Maxwell body, as an example.

Its equations are:

$$
\begin{align*}
e &= \zeta + p, \\
s &= K \zeta + V \dot{p}
\end{align*}
$$

(6)

as represented by the opposite rheological model.

It is supposed that both $K(x)$ and $V(x)$ are uniformly upper and lower bounded on $[0, 1]$.

§ 2. Periodic constitutive equations and functional framework

Let us suppose that $\varepsilon_0$ and $f^0$ are $T$-periodic time functions