Ordering Processes in Frustrated Systems

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Abstract

Ordering processes in frustrated systems are studied mainly on spin systems on lattices. The temperature dependence of the degree of order, nature of phase transitions and dynamic properties of ordering processes are studied on frustrated lattices. A discussion on the glass state is given by taking into account frustrated situations in the glass state.

§1. Introduction

Ordering processes have been studied with great interest in statistical physics, in particular, in lattice spin systems whose Hamiltonians are given in the form:

\[ \mathcal{H} = - \sum_{i,j} J_{ij} S_i S_j , \]  

(1.1)

where \( J_{ij} \) is a coupling constant given by the exchange integral and \( S_i \) denotes a spin on the \( i \)-th site. Rigorous results have been obtained in many models [1] and also various methods such as the mean field approximation [2], series expansion [3] and renormalization method [4] have been developed to understand ordering processes. We have had considerably good understanding on ordering processes of so-called pure models where the ground state is almost uniquely given (at most it has trivial degeneracy).

Recently, random spin systems have aroused interest [5] and it has been pointed out that we have to study not only pure models but also a new category of models, that is, models with frustration [6]. On the frustrated lattices we cannot arrange spin configurations to satisfy all interactions (bond configurations) as shown in Fig.1.

Although the ordering process in the pure system is understood as a competition between the energy (interaction) and the entropy (temperature), we find competition even in interactions in the frustrated systems and the ordering processes on it become complicated.
When we study topologically disordered systems, we can point out one of the most characteristic properties of those systems that local orders make up kinds of global discrepancy [40]. This property will be expressed by using a frustrated lattice. Further, we could say that frustrated lattices are topologically different from the pure one [7] if we introduce the following classification. We classify bond configurations \( \{J_{ij}\} \) according to an equivalence which identifies configurations if one of them can be transformed to another by local gauge transformations

\[
(S_i, J_{ij}) \rightarrow (-S_i, -J_{ij} \text{ for all } j),
\]

as shown in Fig.2.

Thus we expect that we can understand important features of topologically disordered systems by studying frustrated systems. One of the most important properties of frustrated lattices is the existence of degeneracy of (local) stable states [8], in other words, the existence of metastable states, which causes the thermodynamic properties [8-10] and also dynamic properties to change from the pure ones in various ways which will be studied in the following sections (see Fig. 3).

In §2, frustrated Ising models with nearest-neighbor interactions are studied. In §3, fully frustrated models with extra interactions which cause nonzero critical temperatures are discussed. In §4, a discussion including consideration of the glass state is given.

\begin{align*}
\hat{H} &= \sum_{<ij>} J_{ij} \sigma_i \sigma_j, \\
&= (1.2)
\end{align*}

Fig.3. a) A typical example of degeneracy. The symbol \( \blacktriangle \) denotes a frustrated plaquette. The two configurations b) and c) are degenerate as the ground state, where \( \circ \) and \( \bullet \) denote up and down spins, respectively. b) Bold lines are frustrated. c) Shaded lines are frustrated (cited from [8]).

§2. Frustrated Ising Systems with Nearest Neighbor

In this section we will study the thermodynamic properties and also dynamic properties of frustrated Ising systems with nearest-neighbor interactions:

\[
\mathcal{H} = \sum_{<ij>} J_{ij} \sigma_i \sigma_j,
\]

(2.1)