12. Quantum Chromodynamics. Perturbation Theory

In quantum chromodynamics, hadrons are considered to be bound-states of quarks. Interactions with a hadron reduce to interactions with its quark constituents. Strong interactions between quarks are mediated by gluons — the gauge fields of the colour group $SU_3$. The coloured $SU_3$-symmetry is considered exact, and therefore the gluon mass is zero.

At present there exist two main lines in investigating the quantum chromodynamics: (i) perturbative approach, based on perturbation theory, and (ii) non-perturbative approach, which does not use perturbation theory.

In this chapter we shall construct the covariant perturbation theory for quantum chromodynamics. The applications of this theory will be illustrated by several examples. Finally, the basic methods of quantum chromodynamics will be presented which make it possible to take into account the contribution of the gluon effects to the structure functions.

The non-perturbative version of quantum chromodynamics, based on approximating the continuous space-time by a discrete lattice of finite size, will be dealt with in the next chapter.

12.1 Covariant Perturbation Theory for Quantum Chromodynamics

12.1.1 The Lagrangian for Quantum Chromodynamics

Consider the $SU_3$-triplet of spinor fields

$$\psi^a(x) = \begin{pmatrix} \psi^1(x) \\ \psi^2(x) \\ \psi^3(x) \end{pmatrix}.$$  \hspace{1cm} (12.1.1)

The free Lagrangian for such a triplet is written in the form

$$L(\psi^a(x), \partial_\mu \psi^a(x)) = i \bar{\psi}^a \gamma_\mu \partial_\mu \psi^a - M \bar{\psi}^a \psi^a.$$ \hspace{1cm} (12.1.2)

It is invariant under the global non-Abelian group of $SU_3$-transformations in a space referred to as the colour space:

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12.1 Covariant Perturbation Theory for Quantum Chromodynamics

\[ \psi^a(x) \rightarrow \psi'^a(x) = \left[ \exp \left( -i \frac{g}{2} \lambda_m \epsilon_m \right) \right]_{ab} \psi^b(x), \]

\[ \bar{\psi}^a(x) \rightarrow \bar{\psi}'^a(x) = \bar{\psi}^b(x) \left[ \exp \left( i \frac{g}{2} \lambda_m \epsilon_m \right) \right]_{ba}. \]

Here \( \epsilon_m \) are the (constant) parameters of the group \( SU_3 \), \( \lambda_m \) are the Gell-Mann matrices defined by (1.2.13); \( g \) is the coupling constant.

According to (12.1.3) we have for the infinitesimal transformations of functions:

\[ \delta \psi^a(x) = -i \frac{g}{2} (\lambda_m)^{ab} \epsilon_m \psi^b(x), \]

\[ \delta \bar{\psi}^a(x) = i \frac{g}{2} \bar{\psi}^b(x) (\lambda_m)^{ba} \epsilon_m, \]

or for the generators of the transformations

\[ T^{k}_{ab} = -i \frac{g}{2} (\lambda_k)_{ab}. \]

Taking into account (1.2.14) we find

\[ [T^k, T^l]_{ab} = -\frac{g^2}{4} [\lambda_k, \lambda_l]_{ab} = gf_{klm} T^m_{ab}. \]

Let us now turn to the local group of \( SU_3 \)-transformations. According to (2.1.12), the covariant derivative has the form

\[ \nabla_{\mu} \psi^a(x) = \partial_{\mu} \psi^a(x) + i \frac{g}{2} (\lambda_m)^{ab} \psi^b(x) V_{\mu}^m(x). \]

As can be seen, in this case the octet of the vector fields \( V_{\mu}^k(x) \) is gauged. These fields are called the gluon fields.

The Lagrangian for the gluon fields reads, by virtue of (2.2.23, 24),

\[ \mathcal{L} = -\frac{1}{4} F^{k}_{\mu\nu} F^{k}_{\mu\nu}, \]

where

\[ F^{k}_{\mu\nu} = \partial_{\mu} V_{\nu}^k - \partial_{\nu} V_{\mu}^k - \frac{g}{2} f_{n pk} (V_{\mu}^n V_{\nu}^p - V_{\nu}^n V_{\mu}^p) \]

is the gluon field tensor.

The gluon fields transform, according to (2.2.7), as follows:

\[ \delta V_{\mu}^m(x) = g f_{n pm} V_{\mu}^p(x) \epsilon_n(x) + \partial_{\mu} \epsilon_m(x). \]

For the total locally invariant Lagrangian we have