3. Spontaneous Symmetry-Breaking

As we have seen in Chap. 2, the gauge-invariant theories include massless gauge fields. From the viewpoint of physical applications both massless and massive gauge fields are relevant. Just adding a mass term to the Lagrangian for the gauge field is not allowed since it would lead to the violation of the gauge-invariance of the Lagrangian. Therefore, a different approach has been proposed in which gauge fields acquire a mass by breaking of the gauge-invariance of the vacuum, while the Lagrangian of the gauge field remains gauge invariant (spontaneous symmetry-breaking). Symmetry-breaking of the vacuum may be incomplete. A part of the gauge fields then remains massless. This makes it possible to build theories including both massive and massless gauge fields, a circumstance used in unifying the short-range and the long-range interactions (e.g., the weak and the electromagnetic interactions), whose mediators are massive intermediate bosons and massless photons, respectively.

First we shall explain what spontaneous symmetry-breaking is and then consider the mechanism of spontaneous breaking of the global and the local invariance emphasizing their specific features. Finally, we shall dwell on the residual symmetry.

3.1 Degeneracy of the Vacuum States and Symmetry-Breaking

Consider a quantum-mechanical system. Let it be described by the Lagrangian $L$ or the Hamiltonian $H$. The system can be in various energy states $E_n$ determined by the equation

$$H \psi_n = E_n \psi_n.$$ 

Each state is specified by a certain value of the energy $E_n$ and by the wave function $\psi_n$. The state of minimum energy $E_0$ described by the wave function $\psi_0$ is called a vacuum one. If a single vacuum state corresponds to the value $E_0$ it is called a non-degenerate vacuum state, otherwise it is called degenerate.

Let a definite transformation group $G$ be given. The vacuum state is invariant under the group $G$ if it transforms into itself and non-invariant otherwise.

In the framework of local relativistic quantum field theory there exists a connection between the invariance of the vacuum state under a group of trans-
3.2 Spontaneous Breaking of Global Symmetry

3.2.1 Exact Symmetry

Consider a model described by the Lagrangian

\[ L = (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2 \phi^* \phi - \frac{1}{4} f(\phi^* \phi)^2, \tag{3.2.1} \]

where \( \phi(x) \) is a complex scalar field, \( f \) is the coupling constant of the scalar fields, \( f > 0 \), and \( m \) is the mass of the scalar particle, \( m^2 > 0 \).

This Lagrangian is invariant under the global group \( U_1 \) of the phase transformations

\[ \phi(x) \rightarrow \phi'(x) = e^{-ige} \phi(x), \quad \phi^*(x) \rightarrow \phi'^*(x) = e^{ige} \phi^*(x). \tag{3.2.2} \]