I. INTRODUCTION.

In [8] we introduced a class of image models for various tasks in digital image processing. These models are multi-level or "hierarchical" Markov Random Fields (MRFs). Here we pursue this approach to image modelling and analysis along some different lines, involving segmentation, boundary finding, and computer tomography. Similar models and associated optimization algorithms appear regularly in other work involving immense spatial systems; some examples are the studies in these proceedings on statistical mechanical systems (e.g. ferromagnets, spin-glasses and random fields), the work of Hinton and Sejnowski [14], Hopfield [15], and von der Malsburg and Bienenstock [19], in neural modeling and perceptual inference, and other work in image analysis, e.g. Besag [2], Kiiveri and Campbell [17], Cross and Jain [5], Cohen and Cooper [4], Elliott and Derin [7], Deviver [6], Grenander [11], and Marroquin [20]. The use of MRFs and related stochastic processes as models for intensity data has been prevalent in the image processing literature for some time now; we refer the reader to [8] and standard references for a detailed account of the genealogy.

The aforementioned analogy between very large (usually spatial) stochastic systems such as those encountered in digital image processing, computer vision, and neural modelling, and the lattice-based systems of statistical mechanics has been an important theme of our past work. For instance, our computational algorithms are based on a new optimization technique called "simulated annealing", introduced by Černý [3] and

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Kirkpatrik et al [18]. Stochastic relaxation and simulated annealing are briefly discussed in § V and remain the basis of our reconstruction and segmentation algorithms. However, the focus here is on image modelling, statistical inference, and new applications.

Our image models are "hierarchical" and stochastic. First, we regard the "image" as a collection of attribute processes, only one of which is the usual array of intensity or brightness values. The other, mainly geometric, attribute processes are constructs, corresponding to edges, object locations, feature labels, and so forth; they are part of the image model but not of the physical data. We use the term "hierarchical" to reflect the fact that image attributes such as boundaries and texture labels involve increasingly global and contextual information and expectations.

We have chosen the family of MRF priors for "images" for several reasons. First, we believe this formulation provides a solid, theoretical basis for complex image modelling: the class of models is extremely rich and easily accommodates a multi-level framework. Indeed, spatially-invariant, geometric attributes such as edges, curves, and simple polygons (with arbitrary scale and location) can be incorporated in the model in a local fashion. This was illustrated in [8] with the addition of a "line process". Second, the duality between MRFs and Gibbs distributions (see § II) allows the modelling process to be explicit and constructive: we build energy functions to quantify our a priori expectations about imagery. Finally, for many types of degradations (see § III), the conditional independence (= Markov property) of the prior is inherited by the posterior distribution. This is crucial because it guarantees a satisfactory degree of computational feasibility; see §§ IV, V.

For many problems in "low-level" image processing and related fields the current models appear adequate; other needs are more pressing, such as reducing the computational load and developing a rational data-driven method for estimating parameters in the model (see § VI). It remains to be seen whether the hierarchical MRF framework can accommodate the necessary high degree of external knowledge to deal with problems in "high-level" vision (for instance object recognition and texture labeling). Basic concepts such as scale and shape must be merged into the graph and model structures, and in a way that is sufficiently local to avoid unrealistic amounts of computation. Some of our preliminary experiments, and those of others, are encouraging. This paper addresses a "middle-level" of problems in reconstruction and segmentation in which excellent results are possible with some degree of "knowledge engineering", coupled with a careful analysis of the degradation mechanism.