Phonon Scattering in Multiquantum Well Structures

M. Singh

Department of Physics, The University of Western Ontario, London, Canada N6A 3K7

A theory of phonon scattering by carriers (electrons, holes) in a quasi two-dimensional gas has been developed for the multiquantum well structures (MQWS) in the presence of a magnetic field. We call it magneto-phonon scattering. The expressions for the Landau-level width (LLW) due to hole-phonon scattering and the cyclotron resonance line width (CRLW) due to electron-phonon scattering have been obtained. We found that the LLW and CRLW increases with the magnetic field and the temperature and decreases with the increase of the periodicity of the MQWS. It is found that the LLW of light holes is higher than the LLW of heavy holes. Finally the screened polar optical phonon contribution to the damping of the magneto-phonon oscillation in the MQWS is also discussed.

INTRODUCTION

The carrier-phonon interaction plays a very important role in the transport properties and the optical properties of quasi two-dimensional MQWS. Here carrier refers to electrons and holes. The electron-phonon scattering in zero magnetic field of a quasi two-dimensional electron gas in the presence of an infinitely deep quantum well (IDQW) has been calculated by several authors, /1/, Prasad and myself/1/ calculated the relaxation time using the Green's function approach for the IDQW. Recently myself and Chaubey/2,3/ have calculated the Landau level width due to electron-phonon scattering and the damping of the magneto-phonon oscillation (MPOO) due to electron-optical phonon scattering in the MQWS. In this paper we turn our attention to the calculation of the Landau-level width due to hole-phonon interactions and the cyclotron resonance line width due to electron-phonon interactions. We applied our theory to calculate the LLW and CRLW of a GaAs/GaAl superlattice. It is found that the LLW and CRLW increase with increasing temperature and magnetic field and decreases with an increase of the period of the MQWS. Theoretical results are in qualitative agreement with experimental results.

THEORY

The carrier-phonon interaction Hamiltonian of a two-dimensional carrier gas in MQWS subjected to a static magnetic field B applied perpendicular to the layers is given by

\[ H_{cp} = \sum_{N',x',p',m'} F_{N',x',p',m'} \left( a_{N',x',p',m'}^\dagger \right) a_{N,x,p,m} \right\}

where \( N, p, m \) and \( x \) are the Landau quantum number, subband quantum number, carrier band quantum number and centre of the cyclotron orbit, respectively. \( a_{N,x,p,m}^\dagger \) and \( a_{N,x,p,m} \) are the creation and annihilation operators, respectively, for the carrier in the Landau level \( |\lambda\rangle = |N,x,p,m\rangle \), whereas \( b_{N,x,p,m}^\dagger \) and \( b_{N,x,p,m} \) are the creation and annihilation operators, respectively, for a phonon wave vector...
\( \mathbf{q} \cdot \mathbf{R}(r,z) \) is the position vector of a carrier in MOWS and \( \mathbf{q} = (q_x, q_z) \), \( q_x \) and \( q_z \) being the components of the phonon wave vector along the layer and perpendicular to the layer, respectively. \( i \) stands for phonon polarization. The values of \( V_{q,t} \) for electron-acoustic phonon and electron-optical phonon interactions are given in references /4/ and /5/, respectively. The value of \( V_{q,t} \) for hole-phonon interactions is written as

\[
V_{q,t} = \left[ \frac{3\Omega R_d^2}{24N_t} \right] \left[ \hat{\mathbf{D}} + D_j \mathbf{J}^q \right] q_t^q (q_t) + (j_x j_y + j_y j_x) \left[ \hat{e}_t \right] \hat{q}_y + \hat{e}_t \hat{q}_x + \text{C.P.} \]

Here we consider the Bir and Pikus /6/ type of interaction Hamiltonian which includes the effects of internal strains. \( D_1 = E_a/E_d, \ D_2 = \sqrt{3}(E_b/E_d) \), where \( E_a, E_b \) and \( E_d \) are the deformation potential. C.P. refers to terms obtained through cyclic permutation of indices and \( J_\alpha \) is the \( \alpha \)th component of the angular momentum \( J = 3/2 \). \( \hat{q} \) is the unit vector along the direction of \( q \) and \( \hat{e}_t \) is the polarization vector in the \((q,t)\) mode. The self-energy \( E_\lambda(E) \) of the retarded Green's function \( G_{\lambda}(E) \) describing the effect of the scattering of a carrier in the Landau level \( |\lambda\rangle \) is given by

\[
E_\lambda = E_{\lambda} \pm \frac{1}{\tau_\lambda} \sum_{\lambda'} \left[ \left| \mathbf{W}_{q,t}^{\lambda\lambda'} \right|^2 \left\{ \left( \frac{N_0(Q)}{E_{\lambda} - E_{\lambda} - \hbar \omega - E_{\lambda}'} \right)^2 + 1 + N_0(Q) \right\} \right] \]

\[
\left| W_{q,t}^{\lambda\lambda'} \right|^2 = \left| \langle \lambda' \left| V_{q,t} \right| \lambda \rangle \right|^2 = \left| P_{NN'}(Q) \right|^2 \left| P_{pp'}(Q) \right|^2 \left| \mathbf{C}_{n_0} \right|^2 \left| \mathbf{C}_{n'0} \right|^2 \]

where \( N_0(Q) = \text{exp} \left\{ \frac{\hbar \omega_{q,t}}{k_B T} \right\} - 1 \) and \( E_{\lambda} \) is the energy of the \( |\lambda\rangle \) state.

Rewriting \( E_{\lambda} = \Delta + i\gamma/2 \) in equ. (3) and then calculating the imaginary part (which is the Landau level broadening LLB) of both sides of the same equation, we obtain the following equations for the LLB for the heavy hole \((m_j = \pm 3/2)\), \((\Gamma_{N,0})\) and light hole \((m_j = \pm 1/2)\), \((\Gamma_{N,1})\).

\[
\Gamma_{N,0} = \sum_{N,0} \left( 2N_0(Q) + 1 \right) \left[ \frac{1}{2} \sum_{\lambda} \left| C_{n_0}^{\lambda} \right|^2 T_{n_0}^h \right] \left[ \frac{1}{2} \sum_{\lambda'} \left| C_{n_0}^{\lambda'} \right|^2 T_{n_0}^h \right] \left( \frac{C_{n_0}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_0}^{\lambda'}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_0}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_0}^{\lambda'}}{E_{\lambda} - E_{\lambda}'} \right)^2 \]

\[
\Gamma_{N,1} = \sum_{N,1} \left( 2N_0(Q) + 1 \right) \left[ \sum_{\lambda} \left| C_{n_1}^{\lambda} \right|^2 T_{n_1}^h \right] \left( \frac{C_{n_1}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_1}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_1}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \left( \frac{C_{n_1}^{\lambda}}{E_{\lambda} - E_{\lambda}'} \right)^2 \]

The values of \( J_{NN'}(q_t) \) and \( F_{pp'}(q_z) \) are given in reference /5/, the value of \( c_{q,t}^{\lambda\lambda'} \) will be published elsewhere /5/, it cannot be given here due to the limited space.

Here we have neglected the \( \hbar \omega_{q,t} \) term with respect to \((E_{\lambda} - E_{\lambda}')\) in equ. (4). The expression for LLW of the electron-phonon interaction are given in ref. /4/. The above two equations are coupled in \( \Gamma_{N,0} \) and \( \Gamma_{N,1} \), which can hence be found by self-consistent calculation. Note that the LLW of light holes depends on the LLW of heavy holes and vice versa. This is due to the interaction between the two bands which plays a very important role in the transport and optical properties of the MOWS. The exact calculation of the LLW for the hole-phonon interaction is very difficult because of the complexity of the valence band structure. In the calculation of LLW the interaction between the valence bands appears through \( c_{q,t}^{\lambda\lambda'} \) and the Landau level energy \( E_{\lambda} \). In the present paper we will neglect the interaction between the valence bands and use the effective mass approximation. The result will still show the qualitative (but not quantitative) behaviour. To get quantitative results one should use a 4x4 Luttinger-type matrix Hamiltonian or \( k \cdot p \) Hamiltonian to calculate the Landau level. Work is in progress on this line.