1 Introduction

The structural reliability analysis is formulated based on two fundamental assumptions: (1) the state of the structure is defined in the outcome space of a vector of basic random variables; (2) the structure can be in one of two states, the safe state or the failure state. The boundary between the two states in the outcome space is known as the limit-state surface.

Let the vector $V$ denote the set of basic random variables pertaining to a structure, and assume the joint probability density function (PDF) $f_V(v)$ is known. The basic random variables may include parameters defining loads, material properties, structural geometry, etc.

Failure criteria of structures are usually defined in terms of the basic random variables, $V$, and a load effect vector, $S$, such as stresses and deformations. Then, $V$ and $S$ are related through the mechanical transformation

$$S = S(V)$$

(1)

For all but trivial structures, this transformation is available only in an algorithmic sense. The finite element reliability methods are reliability methods which use finite element analysis to compute the load effect vector. In accordance with the failure criteria, one can formulate a limit-state function such that $g(v, s) > 0$ defines the safe state, $g(v, s) \leq 0$ defines the failure state, and $g(v, s) = 0$ defines the limit-state surface. Then, the probability of failure of the structure is

$$P_f = \int_{g(v, s) \leq 0} f_V(v) dv$$

(2)

In reliability analysis, it is convenient to transform the variables $V$ into the standard normal space through a probability transformation.
\[ Y = Y(V) \]  

(3)

where the elements of \( Y \) are statistically independent and have the standard normal density. Such a transformation is not unique. The selection of an appropriate transformation is based on the distribution of \( V \) [4,8].

Der Kiureghian and Liu [4] suggested a probability transformation which is particularly useful in the finite element reliability methods. In this method, a joint distribution reliability model, originally introduced by Nataf [12], with prescribed marginal distributions and correlation matrix was proposed. The joint PDF of \( V \) is defined such that the variables \( Z = (Z_1, \ldots, Z_n) \) obtained from the marginal transformations

\[ z_i = \Phi^{-1}[F_{V_i}(v_i)] \quad i = 1, 2, \ldots, n \]  

(4)

are jointly normal, where \( F_{V_i}(\cdot) \) denotes the marginal distribution of \( V_i \), and \( \Phi(\cdot) \) denotes the standard normal cumulative probability. Since \( Z_i \)'s are joint normal with zero means and unit standard deviations, it is completely defined by its correlation matrix. The correlation coefficient \( \rho_{Z_i,Z_j} \) of \( Z_i \) and \( Z_j \) can be expressed in terms of the marginal distributions and correlation coefficient of \( V_i \) and \( V_j \) through an integral relation [4]. The transformation to the standard normal space for the above distribution model, then, is given by

\[ y = L_{Z}^{-1}z \]  

(5)

in which \( L_{Z} \) is the lower triangular matrix obtained from the Cholesky decomposition of the correlation matrix of \( Z \).

In the first- and second-order reliability methods (FORM and SORM), one searches for the nearest point on the limit-state surface to the origin in the standard normal space by solving the constrained optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \sqrt{y^T y} \\
\text{subject to} & \quad G(y) = 0
\end{align*}
\]  

(6)

where \( G(y) \) is the limit-state function in the \( y \) space. The optimal solution can be found by the HL-RF method [6,13], which is based the following recursive formula:

\[ y_{k+1} = \frac{1}{|\nabla G(y_k)|^2[\nabla G(y_k)y_k - G(y_k)]} \nabla G(y_k)^T \]  

(7)

where \( y_k \) and \( y_{k+1} \) are the \( y \) at the \( k^{th} \) and \((k + 1)^{th}\) iterations, respectively, and \( \nabla G(y) \) is the gradient of the limit-state function with respect to \( y \). The optimal point, \( y^* \), is called the design point, and the minimum distance, denoted \( \beta \), is called the reliability