Constitutive Equations of Nonstationary Metal Creep

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Summary
Kinematic equations of creep theory have been proposed by Yu.N. Rabotnov in [1]. Attempts of many researches to state constitutive equations for structural parameters caused discrepancy between theoretical and experimental results in the case of variable step load. We propose to use experimental reload data to define the structural parameters. In building this theory we use and extend ideas in [2,3].

Assumptions on creep phenomena in a specimen under torsion and off-loading

The theory of nonstationary metal creep is based on the assumption that creep curves under constant stress can be approximated by the expression

\[ p(t) = V(\sigma)t + a(\sigma)(1 - e^{-mt}) , \]

where \( p \) is the creep strain, \( \sigma \) is the stress, \( V(\sigma) \) and \( a(\sigma) \) are functions of \( \sigma \) defined by experiment, and \( m \) is a constant that is independent of stress. In [3,4] it is shown that the proposed relationship and experimental data are in good coincidence for metal.

If on the stationary curve segment, stress is suddenly relieved, then reset strain is observed, which terminates in a time interval equal to load time. The maximum magnitude of the reset strain

\[ p(t_1) - p(\infty) = C(\sigma) , \]

is independent of the stress-relief time \( t_1 \) on the stationary curve segment and is a function of initial stress \( \sigma \). The relationship was verified in V.I. Gorelov’s experiments for many metals.

Constitutive equations for simple tension

We will construct the simplest version of the kinetic creep theory to describe exactly the experimental relationship (1) and (2).
Kinetic theory equations with one structural parameter will take the form

\[
\begin{align*}
\dot{p} &= a_1(\sigma) p + b_1(\sigma) q + C_1(\sigma), \\
\dot{q} &= a_2(\sigma) p + b_2(\sigma) q + C_2(\sigma).
\end{align*}
\]

(Eq. 3)

Eqs. (3) for fixed stress are linear. The basic nonlinear effects are described by using \( a_i(\sigma) \) and \( b_i(\sigma) \). To describe relationship (1) and (2), equations (3) become

\[
\begin{align*}
\dot{p} &= q(\sigma) C^{-1}(\sigma) q + V(\sigma) + ma(\sigma), \\
\dot{q} &= -mg - m^2 C(\sigma).
\end{align*}
\]

(Eq. 4)

The initial state \( p(0) = q(0) = 0 \). For \( \sigma = 0 \), the equality \( a(0)C^{-1}(0) = 1 \) is hold.

As the parameter \( q \) is introduced in a rather formal way, it can be replaced by any parameter \( r \) which is a linear function of \( p \) and \( q \):

\[
r = d_1 p + d_2 q, \quad d_i = \text{const}.
\]

(Eq. 5)

It is natural that the transformation (5) does not change the meaning of physical phenomena under description. We will use the relationship (4) in our research. These relationship include three experimental functions \( a(\sigma), b(\sigma), C(\sigma) \) and a constant \( m \). To construct these functions we have to make several experiments for measuring creep curves for different values of \( \sigma \) and estimating the magnitude of reset deformation on the stationary curve segment. Figure 1 illustrates the geometrical meaning of these quantities. It should be noted that \( a(\sigma), V(\sigma), C(\sigma) \) are estimated from most reliable measurements made on the stationary curve segment.

The comparison of the results obtained by integrating (4) for various step changes of \( \sigma \) with experimental measurements on ZI437B, 40HSN2MWF and 5HNIM steels is made in [3,4,6]. At the beginning creep curves for five fixed stress values have been built. Using the curves, we have determined \( a(\sigma), V(\sigma) \) and \( m \) as well as \( C(\sigma) \) associated with the stationary curve segment obtained after relieving the load.