Summary. Stability of stationary motions of an orbital system, including two satellites connected by an elastic massive tether, is studied. Radial equilibrium configurations of the system in the gravitational field are stable. Aero- and electrodynamic forces turn to be responsible for the flutter-type instability of inclined configurations. Internal friction introduces natural damping into the system. Certain restrictions on operational characteristics of the system arise from the stability conditions.

1. Introduction. After an impressive flow of innovative ideas and consequent profound studies by many investigators, space tethers have been eventually recognized as a highly promising new space technology. The first mission of the Shuttle-based Tethered Satellite System (TSS) is scheduled by NASA for 1991. During this flight a half-ton subsatellite will be deployed upwards from the Shuttle Orbiter on a 20 km conducting tether to carry out electrodynamic experiments. During the second TSS mission the subsatellite will be deployed Earthwards to the altitude of 120 km to accomplish aerodynamic measurements.

The effect of electro- and aerodynamic forces on the TSS stability will be the main problem under consideration in this study.

2. Equations of motion. Let the Shuttle Orbiter follow a circular orbit. Motion of the subsatellite \( A \) relative to the orbital frame attached to the center of mass \( O \) of the Orbiter is governed by the ordinary differential equation

\[
m_A \left[ \ddot{r}_A + 2\Omega \times \dot{r}_A + \Omega \times (\Omega \times R_A) + \mu R_A R^{-3}_A \right] = -T_A + F_A,
\]

while motion of a massive and extensible tether is determined by the partial differential equations

\[
\rho \left[ \ddot{R} + 2\Omega \times \dot{R} + \Omega \times (\Omega \times R) + \mu R R^{-3} \right] = T' + F,
\]

\[
T = eE(\gamma - 1).
\]
Here \( \mathbf{r} \) is the radius vector in the orbital axes, \( \mathbf{R} \) the geocentric radius vector, \( \Omega \) the orbital angular rate, \( \mathbf{T} \) the tether tension force, \( E \) the axial stiffness of the tether, \( m_A \) the subsatellite mass, \( \mu \) the Earth’s gravity constant, \( \mathbf{F} \) and \( \mathbf{F}_A \) represent disturbing forces, dots indicate differentiation with respect to time, while primes denote differentiation with respect to a natural parameter \( s \) measured from the Shuttle towards the subsatellite, \( 0 \leq s \leq l \), \( l \) is the total length of the unstressed tether, \( \gamma = |\mathbf{r}'| \) the elongation of the tether, \( \mathbf{e} = \mathbf{r}'/\gamma \) the unit tangent vector.

The tether supporting the subsatellite is attached to the Shuttle, that means

\[
\mathbf{r}_0 = 0. \tag{3}
\]

3. Stability of the radial equilibrium. If no perturbations (\( \mathbf{F} = 0, \mathbf{F}_A = 0 \)), the TSS will have two equilibrium positions relative to the orbital frame: one, with the tether stretched upwards, and the second, with the tether deployed downwards. Equilibria with a slack tether are excluded since they proved to be unstable.

For \( \mathbf{F} = 0, \mathbf{F}_A = 0 \), equations admit of the energy integral

\[
K + W = h \tag{4}
\]

where

\[
K = \frac{1}{2} m_A \mathbf{r}_A^2 + \frac{1}{2} \int_0^l \rho \mathbf{r}'^2 \, ds,
\]

\[
W = \frac{1}{2} \int_0^l E(\gamma - 1)^2 \, ds - \int_0^l \rho \left[ \frac{1}{2} (\Omega \times \mathbf{R})^2 + \frac{\mu}{R} \right] \, ds - m_A \left[ \frac{1}{2} (\Omega \times \mathbf{R}_A)^2 + \frac{\mu}{R_A} \right].
\]

The potential energy \( W \) was found to be positive definite in the vicinity of the radial equilibrium under the condition

\[
E > E_\ast \approx T_A \tag{5}
\]

where \( T_A \) is the equilibrium tension at the end \( A \). Actually, we always have \( E \gg T_A \) since an acceptable strain of any real tether is rather small: \( \varepsilon = \gamma - 1 = T/E \ll 1 \). Therefore, the radial equilibrium is stable in the Lyapunov’s sense.

4. Natural modes of in-plane oscillations. Small oscillations around the radial equilibrium in the orbital plane are governed by the equations

\[
\rho (\delta \ddot{x} - 2 \Omega \delta \dot{y} - 3 \Omega^2 \delta x) = (E \delta x')',
\]

\[
\rho (\delta \ddot{y} + 2 \Omega \delta \dot{x}) = (T \gamma^{-1} \delta y')',
\]

\[
m_A (\delta \ddot{x}_A - 2 \Omega \delta \dot{y}_A - 3 \Omega^2 \delta x_A) = -(E \delta x')_A,
\]

\[
m_A (\delta \ddot{y}_A + 2 \Omega \delta \dot{x}_A) = -(T \gamma^{-1} \delta y')_A,
\]

\[
\delta x_0 = 0, \quad \delta y_0 = 0,
\]

\[
\delta x_0 = 0, \quad \delta y_0 = 0, \tag{6}
\]