THEORETICAL AND LINGUISTIC METHODS FOR DESCRIBING STRAIGHT LINES.

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ABSTRACT

Two distinct approaches exist for the generation of a straight line in an arbitrary direction on a lattice, structural algorithms and conditional algorithms. A survey of these approaches is included in this paper.

Structural algorithms are of great theoretical value. From number theory the Farey-series can be used for directions on a lattice with a rational slope. With continued fractions it is also possible to approximate irrational numbers. Knowing these properties it is possible to determine the set of all lines corresponding to a given chaincode for a segment of a line. This is useful for the estimation of properties of a line like length, slope and intercept. Research of this kind is related to pattern analysis and picture processing.

The structural algorithm can be presented by linguistic methods, for instance a context-free programmed grammar and a TOL-grammar, a variant of the Lindenmayer grammar with the important property that in each derivation each relevant symboltype in a string is rewritten at the same time by the same rule.

The principle of the conditional algorithm is more practical: given two points that determine the line, connected points on the lattice with minimal distance to the real line have to be selected. This method is very important for plotters and computer graphics displays.

Structural and conditional methods both generate lines satisfying the chord property, which is a conditional property. A structural property of a digitized straight line is spacing the least occurring type of chainelement as uniformly as possible. It is shown that this can be built into a conditional method. So an integration between both methods is achieved.

Finally some remarks on progress in the science of line drawing are made.
1. INTRODUCTION

As a student at the Delft Technical University in the Netherlands, I had the opportunity to explore the possibilities of linguistic methods for pattern recognition in addition to the statistical methods. After preparing a survey on this topic (in Dutch) I found for my masters thesis (1972) an application in the generation of straight lines on a lattice by linguistic methods. A publication in English followed in 1974 [4].

In [4] two different approaches for straight line generation are developed, a structural method and a conditional method. Both methods have been presented as algorithms and as linguistic methods. The essential difference is that on the one hand the structural method is based on theoretical reflections on the digitization of a straight line on a lattice, which leads to number theoretical conceptions like Farey series and continued fractions [13]. On the other hand the conditional method is very pragmatic. During the generation of a line from a certain point after each detection of a point on the lattice belonging to the digitized line, there is a test whether a condition has been fulfilled. The result of the test determines next point on the lattice. For a long time the leading conditional method was given by Bresenham in 1965 [21].

In chapter 2 of this paper chain coding and the concept of straightness are examined.

In chapter 3 a survey of the structural method and recent developments is given, and in chapter 4 the same for the conditional method.

After ten years it is very interesting to see that there was almost no communication between structural and conditional describers. Structural describers are mostly found in the world of picture processing and pattern recognition. Conditional describers are especially developing algorithms to drive plotters or graphic displays.