METHODS FOR DRAWING CURVES

K.W. Brodlie
Computer Laboratory
University of Leicester
Leicester, UK.

1. INTRODUCTION

Curve drawing is a fundamental aspect of computer graphics. It occurs in a great variety of different applications. The scientist or engineer, having observed or calculated a sequence of data values, will frequently wish to display an estimate of the function underlying these values by drawing a curve through the data points. If the data points are known to be subject to error, the problem changes to one of drawing a smooth curve which best approximates the data. The cartographer may use curve drawing to depict the boundary of a region, given only a discrete set of boundary points. The designer of a ship or motor car, starting from some initial trial curve will wish to transform it interactively into some desired shape. The animator too will make use of curve drawing in the production of cartoons - not only for pictures on individual frames, but also to construct automatically 'in-between' frames, where corresponding points on successive 'key' frames are interpolated by a curve to give a smooth transition.

In all these applications the problem of curve drawing has two quite distinct aspects. First a mathematical representation of the desired curve must be constructed; then a means is needed of displaying that mathematical representation as a smooth-looking curve on a graphical display surface.

It is perhaps best to begin, therefore, by formulating in mathematical terms the various curve drawing problems described above.
(i) Single-valued curve interpolation

Suppose a scientist collects data values \( (x_i, y_i) \), \( i=1,2,...,n \), where the values \( y_i \) can be regarded as realisations of some underlying function of the independent variable \( x \). The scientist wishes to construct some interpolant \( f(x) \) such that

\[
f(x_i) = y_i, \quad i = 1,2,..,n.
\]

The curve \( f(x) \) is displayed as an estimate of the underlying function. The term 'single-valued' is used to highlight the fact that the underlying function is a single-valued function of \( x \) (i.e. it does not loop back on itself) and this property must be preserved by the interpolant.

(ii) Single-valued curve approximation

If the data values are known to be in error, the scientist requires an approximating function \( f(x) \) such that

\[
\| f(x_i) - y_i \|
\]

is minimised, where \( \| . \| \) is some suitable norm.

(iii) Parametric curve interpolation

The cartographer with a set of boundary positions \( (x_i, y_i) \), \( i = 1,2,..,n \), has to construct a smooth curve through the data points. But this time the curve may be multivalued or even closed. In this case it is convenient to construct a parametric function \( (x(t), y(t)) \) such that

\[
x(t_i) = x_i; \quad y(t_i) = y_i, \quad i = 1,2,..,n
\]

The values of \( t_i \) can be assigned in some convenient way, the simplest being \( t_i = i, \quad i = 1,2,..,n \). Parametric curves have the flexibility of being multivalued or closed if appropriate - just the flexibility that is not wanted in the single-valued case.

In the computer animation example, suppose the animator has drawn a