6.1 Introduction.

The purpose of this chapter is to illustrate the capabilities of the present numerical technique for calculating wave transformation problems. The emphasis will be on the unified treatment of the flow before, during and after breaking. Several typical wave transformation problems were selected, and the results of the computations will be presented by increasing order of complexity. From the viewpoint of surf-zone hydrodynamics, the complexity of the physical phenomena increases from pre-breaking, to transitional (overturning), and finally to broken (turbulent) motion. The numerical simulations reported below are ordered according to this sequence.

First, pre-breaking phenomena are considered, by studying the propagation and collision of solitary waves (§§ 6.2–6.3). The numerical solutions showed that the present model describes accurately the evolution of highly nonlinear waves when nonlinear steepening and dispersivity are in balance. It was found that the order of the theory used for generating initial and boundary conditions has a greater influence in the quality of the simulations than the truncation errors. Second, transition from dispersive to dissipative regime was investigated using the moving hydraulic jump as the basic model (§ 6.4). This study was divided in three stages—undular, transitional and turbulent jumps—corresponding to increasing Froude numbers and depth ratios. It was shown that the three types of jump have distinctive characteristics, which in turn are similar to those of pre-breaking, overturning, and broken waves. The properties of undular jumps were predicted with great accuracy. The computed critical depth ratio was found to agree with the experimentally observed value, and the nature of the transition was explained in terms of the structure of the velocity field. The main features of the turbulence field in turbulent jumps were accurately reproduced, except very near the jump’s front and close to the bottom.

Finally, the typical sequence of wave transformation phenomena across a surf-zone was simulated using both solitary and periodic waves (§ 6.5–6.6). A qualitative and quantitative discussion of the numerical solutions was made using the experimental results of Nadaoka (1986) and Mizuguchi (1986).

6.2 Propagation of a solitary wave over a horizontal bottom.

The propagation of a solitary wave over a horizontal bottom is a classic problem for testing nonlinear models. The accuracy of the model can be evaluated by comparing the
computed celerity and amplitude of the solitary wave with their theoretical values, and by observing the deformation of the original wave during the propagation. In the numerical simulations reported below, wave theories of first, second and third order were used. The effects of the order of approximation, truncation errors and F-convection algorithm will be discussed.

The physical parameters of the problem were chosen as follows. First, time and length units were chosen so that the acceleration of gravity, $g$, and the still water depth, $D$, were both equal to 1. A solitary wave with height $H=0.4$ and initial crest position $z_{crest}=15.0$ was placed in a domain 40 units long and 2 units high. The bottom boundary was chosen free-slip and the remaining boundaries were specified open. On the free-surface inviscid boundary conditions were used.

In the $x$-direction, the domain was discretized using a uniform mesh of 160 cells with $\Delta x = 0.25$. In the $y$ direction, a variable mesh of 18 cells with minimum spacing $\Delta y = 0.075$ near $y = 1.2$ was used, improving the resolution near the wave crest. The time step was chosen $\Delta t = 0.025$.

Figure 6.1 shows the computed velocity field and free-surface configuration for a third order solitary wave (Fenton, 1972), at the time instants $t=0$ and $t=15$. Figure 6.2 illustrates the vertical velocity contours for the same wave at time instants $t=10$ and $t=15$. These contour plots of vertical velocity show very clearly the position of the crest and the deformations suffered by the wave during the propagation. These deformations were a slight steepening of the front face, and small dispersive losses.

Table 6.1 shows the computed crest position $z_{crest}$, its theoretical value $z_0 + ct$, and the computed wave amplitude $H$, for first, second and third order waves, at several time instants. The theoretical celerity was calculated using the dispersion relations appropriate for each order of approximation. For the first-order theory, the computed wave became higher and steeper and showed pronounced dispersive losses, but the computed celerity coincided with the theoretical value to within four significant digits. For the second order theory, the computed celerity was 4.2% higher than the theoretical value and the amplitude remained equal to its initial value, but the wave became progressively more asymmetrical (for $t=15$ it had a nearly vertical front). For the third-order wave, the computed celerity was 2.0% higher than the theoretical value, the amplitude after 15 time units was equal to the initial value, and the deformations and dispersive losses were smaller than in the previous cases.