The Pairing Theory –
Its Physical Basis and Its Consequences

J.R. Schrieffer
Department of Physics, University of California,
Santa Barbara, CA 93106, USA

ABSTRACT: The key developments which set the scene for the microscopic theory of superconductivity are discussed and the physical reasoning which lead to the pairing theory is presented. Consequences of the BCS theory are reviewed.

I. INTRODUCTION

Following the discovery of superconductivity in 1911, a 50-year search for the proper microscopic theory of this phenomenon was pursued by many of the foremost theoretical physicists at that time. To put in proper perspective the final resolution of this problem, it is helpful to recall a few key steps along the way.¹

For two decades after its discovery, superconductivity was considered, in essence, to be a state of zero electrical resistance. That is, if an electric current is set up in a loop, the current will circulate indefinitely without dissipation exhibiting electrical perpetual motion. Not until 1933 with the discovery of the Meissner effect did it become evident that a totally different phenomena was actually involved, namely that of perfect diamagnetism. The essential point is that the currents which flow in a superconductor are very much like the diamagnetic current which flows in the ground state of a hydrogen atom when it is placed in a magnetic field, namely, the 1s wave function is essentially, unaltered by the magnetic field and currents flow in the 1s orbital is due to the A term not the P term in the current density.

These diamagnetic currents shield the external field only weakly in the hydrogen atom, but the shielding is nearly perfect in a bulk superconductor. When the magnetic field is removed the currents are again reduced to zero. This Meissner state or state of flux expulsion is the lowest free energy state of a bulk superconductor so long as the magnetic field is less than a critical field $H_C$. We now know that a state intermediate between this Meissner state and the normal state can be achieved
in so-called type II superconductors with the magnetic field entering as quantized flux tubes which occupy only a portion of the volume, thereby leading to coexistence of magnetic fields and superconductivity in a bulk superconductor.

Soon after the discovery of perfect diamagnetism, Hans and Fritz London advanced a phenomenological relationship between the current density $j$ and the magnetic vector potential $A$, now known as London’s equation,

$$\vec{j}(\vec{r}) = -c\Lambda \overrightarrow{A}(\vec{r}). \quad (1)$$

The constant $\Lambda$ is a phenomenological parameter which is related to the magnetic penetration depth $\lambda$ in the superconductor by

$$\lambda^2 = 4\pi\Lambda, \quad (2)$$

which follows by combining London’s equation with the last Maxwell equation

$$-\nabla^2 A = \frac{4\pi j}{c}. \quad (3)$$

Fritz London quickly suggested a possible quantum mechanical interpretation of equation 1. He proposed that if the ground state wave function of the superconductor is essentially unchanged by the application of the magnetic field (so-called London rigidity), then the London equation follows automatically. This can be readily seen by noting that if $\Psi_B \simeq \Psi_0$ then the current density satisfies

$$\langle j_B \rangle_B = -\frac{\rho_s e}{m} \left\langle \vec{p} + \frac{e \overrightarrow{A}}{c} \right\rangle_0 = -\frac{\rho_s e^2}{mc} \overrightarrow{A}, \quad (4)$$

where $\rho_s$ is the electron density in the superfluid. One can view the electrons in the superconductor as divided into two copenetrating fluids, one of superfluid density $\rho_s$ and the other a normal fluid of density $\rho_n$ having the traditional properties of electrons in a normal metal. Thus the total electron density is divided as

$$\rho = \rho_s + \rho_n, \quad (5)$$

where $\rho_s$ vanishes at the transition temperature $T_c$ and $\rho_n$ vanishes at zero temperature. Combining equations (2) and (4) with $\rho_s = \rho$ at low temperature one obtains fairly good agreement with the observed magnetic penetration depth at low temperature in both conventional superconductors and particularly in high $T_c$ materials.