O-D DEMAND ADJUSTMENT PROBLEM WITH CONGESTION: PART I. MODEL ANALYSIS AND OPTIMALITY CONDITIONS

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The problem of adjusting (or estimating) an origin-destination (O-D) matrix by using observed flows on the links of a congested traffic network, which we denote DAP, is considered in this paper. After reviewing the previous contributions made in stating models and development solution algorithms for this problem, a nonlinear bilevel programming formulation is proposed to model the DAP. The existence of solutions is proved under relatively mild assumptions on the link cost functions and the property of the continuous dependence of equilibrium link flows on the demand is demonstrated under a fairly weaker condition. By using the general bilevel programming theory, the DAP is reformulated as a single-level like optimization problem, where the marginal function of the lower level equilibrium problem is used explicitly in a constraint. The gradient function of the implicit marginal function is derived in terms of the link cost mapping and the link proportions in an equilibrium state. Necessary optimality conditions for the DAP are derived based on the gradient information of the marginal function, of which the significance and application for the DAP are discussed as well.

1. Introduction and Review

The travel demand between origins and destinations on a transportation network is an essential component of network models used in transportation planning. It is a basic input to all network assignment methods both in static and dynamic contexts. The conventional methods of obtaining origin-destination (O-D) matrices are based on the results of surveys, which are costly, or by using various demand modeling schemes. Link counts of vehicular traffic, which are relatively easy to obtain, provide information which may be used to
adjust, or estimate, known O-D matrices which are out of date. The use of the information contained in these link counts has motivated the development of various methods for adjusting O-D matrices. In a broad sense, the O-D demand adjustment problem (DAP) was defined by Cascetta and Nguyen [10] as that of "determining an estimate of the O-D trip demand by efficiently combining traffic count data and all other available information". Before proceeding with a brief literature review, the notation used in this paper is introduced next.

Let $R = (N, A)$ be a transportation network, where $N$ is the set of nodes and $A$ the set of links. Denote by $I$ the index set of the O-D pairs of the network, $\nu = (\nu_a; a \in A)$ the arc flow vectors, $h=(h_k; k \in K_i, i \in I)$ the path flow vectors, where $K_i$ refers to the set of all paths between O-D pair $i$, and $g = (g_i, i \in I) \in G$ the demand vectors (matrices) for all O-D pairs, where $G$ is a bounded set. The average cost of transportation on arc $a$ is denoted by $s_a(\nu)$ while the average cost of transportation on path $k$ by $s_k(h)$. The cost functional of the network is then defined by $s(\nu) = (s_a(\nu); a \in A)$ in terms of arc flows or $s(h) = (s_k(h); k \in K_i, i \in I)$ in terms of path flows. Let $P(g): G \rightarrow I \times A$ be a proportion matrix map, where the cells $p_{ia}(g)$ are the proportions which assign demand $g_i$ to arc $a$.

One of the first mathematical formulations of the DAP is the following. It is assumed that observed link counts $\hat{\nu}_a$ are available for a subset of arcs $\hat{A}, \hat{A} \subseteq A$. The problem is to find $g^* \in G$ such that, when $g^*$ is assigned to the network according to the proportion map $P(g^*)$, the resulting arc flows reproduce the counts on $\hat{A}$, or equivalently, to find a solution to the system of nonlinear equations:

$$\sum_{i \in I} p_{ia}(g)g_i = \hat{\nu}_a, \quad a \in \hat{A},$$

(1)

where $p_{ia}$ is the proportion that demand $g_i$ is assigned to arc $a$.

Different assumptions made for the assignment map $P(g)$ and on the observed link counts result in a large variety of formulations and solution methods for the DAP. When the proportion map $P(g)$ is independent of the demand, such as in network assignment models with