A SYSTEM OPTIMAL TRAFFIC ASSIGNMENT MODEL WITH DISTRIBUTED PARAMETERS

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In this paper a multidestination system optimal dynamic traffic assignment model with distributed parameters is examined. This model can be considered an extension of previous and well known models such as those of Merchant and Nemhauser (1978) and Friesz (1990). Flow dynamics of the model is based on an extension of the simple continuum model for flows composed by several commodities with equal propagation characteristics. An important property of flows following this model is that no overtaking can occur between flows of different commodities. In the case of flow propagating at constant speed and vertical queues at the end of links it is possible the stable approximation of the proposed dynamic system optimal model by means of optimal control problems. For one of the possible stable approximations it is shown how a strengthened Courant-Friedrichs-Levy condition ensures no overtaking and FIFO observance at vertical queues of the approximating optimal control problem. Finally, and for the case of a single destination in the network, the application of an extremals calculation method for optimal control problems developed by the authors in previous papers is also shown and favorable conditions for its application are discussed.

1. Introduction

In the last years dynamic behaviour of urban traffic flows under route guidance systems has been modeled by some authors using a deterministic optimal control approach formulation. This formulation has emerged as a dynamic extension of wardropian equilibrium principles widely used in urban transportation planning. Deterministic optimal control based formulation was stated initially by J.F. Luque and T.L. Friesz [8] (1980) coming from a Merchant and Nemhauser model [9], [10] for networks with a single destination. M. Carey [2], [3]
analyzed the constraints qualification of Merchant and Nemhauser's model. Also models with multiple destinations have been proposed by W. Wie, Y.L. Friesz and R.L. Tobin [15]. All these models had the common characteristic of using the so called exit link functions in order to model traffic dynamics. Models without exit link functions also based on deterministic optimal control have been proposed by B. Ran, D. Boyce and Leblanc L.J. [13]. Although models with and without exit link functions provided a compact formulation they presented some non adequate properties for the evolution of traffic flows. Thus, models without exit link functions presented unproper characteristics when applied to traffic flows and in [7] E. Codina shows paradoxes of the solutions given by dynamic traffic assignment models without exit link functions that lead to an extension of the Dafermos-Sparrow theorem for static traffic assignment models. Instantaneous propagation characteristics of flow dynamics in models with exit link functions are analyzed in [6] and it is shown how this unproper characteristic vanishes as a simple continuum model is approximated. Another unproper characteristic for multideestination models already shown by M. Carey in [3] consists of the conflict arising when flows with different destinations exit from links and the observation of FIFO queueing disciplines by these flows. However, as suggested by Papageorgiou in [12], models using exit link functions can be viewed as an extension of a model with distributed parameters (E. Codina [7]).

This paper presents a general continuous System Optimal Dynamic Traffic Assignment model for multidestination networks. Flows of this model verify the inviscid PDE for stationary and irrotational flows also known as the simple continuum model and this ensures that neither instantaneous propagation nor overtaking may occur between flows of different commodities at links. Also, because of its basic traffic behaviour characteristics, i.e. finite propagation speed, wave dispersion and effects of congestion (i.e. speed reduction and spillback) this model can be taken as a reference. In fact, it is shown how Merchant and Nemhauser's model can be considered a discrete approximation to the continuous system optimal model presented here for the case of density dependent propagation speed and how Friesz's and Wie's models are also approximations by means of continuous time optimal control in the