Chapter 14

Open-Channel Hydraulics

14-1 Uniform Flow, Normal Depth, and Critical Depth

14.1.1 Hydraulic Radius and the Chezy Formula

14-1.1.1 Three kinds of approaches are used to obtain the equation used in open-channel hydraulics. However, all of these mathematical forms originate from the Newtonian equation, and a criterion for choosing one rather than another does not really exist.

Most of the basic equations of this chapter can be obtained from (1) a simplified form of the Eulerian equations in which an empirical friction term is included, (2) the generalized Bernoulli equation in which the condition at the free surface $p = p_a$ (atmospheric pressure) is inserted, or (3) by a direct application of the balance of forces in which the significant terms only are included.

14-1.1.2 Consider a uniform flow parallel to the axis $OX$ (see Fig. 14-1). Since the motion is steady, $\partial(u,v,w)/\partial t = 0$, and since the motion is uniform, $v$ and $w = 0$, and $\partial u/\partial x = 0$. Hence, it is easily verified that all the inertial terms are zero. Moreover, the pressure forces acting on each side of cross

Figure 14-1 Uniform flow.
section \(A\) of an element of fluid \(A \Delta x\) are in equilibrium. The \(OZ\) components of the pressure force and gravity also balance independently of the flow velocity. Thus, the only significant forces which remain are the gravity component in the \(OX\) direction and the shearing forces:

\[ \rho g A \Delta x \sin \theta = \Delta x \int_0^P \tau \, dP \]

where \(P\) is the “wetted perimeter” (i.e., the length of the perimeter of the cross section \(A\) which is underwater) and \(\tau\) is the shearing stress per unit area.

14-1.1.3 In general, \(\tau\) is not constant, except around a circular pipe due to symmetry. However, owing to secondary currents, the variation of \(\tau\) can often be considered as negligible (see Section 8-2.3).

The previous equation is then written

\[ \rho g R_H \sin \theta = \tau \quad \text{where} \quad R_H = \frac{A}{P} \]

Note that the “hydraulic radius” \(R_H\) has the dimension of a length. It is easily verified that in the case of a rectangular section of depth \(h\) and width \(l\)

\[ R_H = \frac{lh}{l + 2h} \]

and \(R_H\) tends to \(h\) when \(l \to \infty\); i.e., in the case of a large river. In the case of a circular pipe of radius \(R\)

\[ R_H = \frac{\pi R^2}{2\pi R} = \frac{R}{2} \]

14-1.1.4 In the case of a river or a channel, the Reynolds number is generally large, and the flow is fully turbulent. The shearing stress can then be assumed to be related to the average velocity \(V\) by a quadratic function such as

\[ \tau = \rho f V^2 \]

where \(f\) is a dimensionless friction factor. When this is combined with the equation \(\rho g R_H \sin \theta = \tau\) the following equation for \(V\) is obtained

\[ V = \left(\frac{g}{f}\right)^{1/2} (R_H \sin \theta)^{1/2} \]

The Chezy coefficient of dimension \((LT^{-2})^{1/2}\) is \(C_h = (g/f)^{1/2}\); thus \(f = g/C_h^2\). The slope \(S\) is generally small, so that \(S = \tan \theta \approx \sin \theta\) and \(V = C_h(R_H S)^{1/2}\). This is the Chezy formula.

The discharge \(Q_n = VA\) is then

\[ Q_n = AC_h(R_H S)^{1/2} = K S^{1/2} \]

where \(K = AC_h(R_H)^{1/2}\) is the conveyance of the channel and depends upon the geometry of the cross section of the channel and the water depth only. \(Q_n\) is the normal discharge, which is defined as a function of water depth for a given channel.

14-1.15 The Chezy coefficient and \(f\) can only be determined by experiment. It is found that in the case of turbulent flow over a rough bottom

\[ C_h = \frac{1.486}{n} R_H^{1/6} \]

where \(R_H\) is in feet and \(n\) is the Manning coefficient; \(n\) is given as a function of relative roughness and in practice varies between 0.01 and 0.05. Inserting this expression in the Chezy formula gives the Manning formula

\[ V = \frac{1.486}{n} R_H^{2/3} S^{1/2} \]

and the conveyance

\[ K = \frac{1.486}{n} A R_H^{2/3} \]

14-1.2 Normal Depth and Transitional Depth

14-1.2.1 The normal depth \(h_n\) is defined as the distance between the lowest part of the channel and the free surface of a uniform flow. It is determined by the equality

\[ Q_n = K(h_n) \]