The Multiple Vehicle DIAL-A-RIDE Problem

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The multiple vehicle many-to-many routing problem is presented in the context of a dial-a-ride system. It is solved by mini-clustering first and optimal routing second. A heuristic mini-cluster algorithm groups together nearby customers who can be efficiently served by the same vehicle route segment. An optimal column generation algorithm then constructs routes corresponding to drivers' workdays by stringing together these vehicle route segments. Problems with up to 200 customers and 85 mini-clusters are easily solved. Larger problems are solved by dividing the day into time slices and applying the algorithm several times.

1 Problem Description

This article examines the use of mathematical optimization algorithms to construct vehicle routes in a large scale multi-vehicle many-to-many system in the context of the transportation of the handicapped. The main contributions of this paper are the innovative concept of the mini-clusters, the generalisation of the column generation algorithm for the m-TSP with time windows (Desrosiers, Soumis and Desrochers [11]) to solve problems with several depots and availability constraints, and in addition, a method of decomposition into time slices to handle very large problems.

For a given day, we have a set of transportation requests, a homogeneous vehicle fleet and a set of pieces of work for drivers. Each transportation request involves: a pick-up stop and a delivery stop, a given number of clients to be transported, a desired pick-up or delivery time, a departure time interval and an arrival time interval, loading and unloading durations.

Vehicles have capacity limits on the number of wheelchairs, and an independent capacity for ambulatory passengers. A piece of work is part or all of a driver workday during which a driver and a vehicle are available to carry out circuits beginning and ending at the same depot. The types of pieces of work available, the number of each type, and the assignment of drivers to vehicles is determined in advance. We consider the case with several depots.

Distances and times between the various stops are given. The problem consists of minimizing the number of pieces of work used and then for this number of pieces of work, minimizing travel time (a function of the distance traveled) while satisfying the following constraints: visiting constraints (each pick-up and delivery stop is visited only once); time window constraints (the time constraints must be satisfied at each stop); capacity constraints (vehicle capacity cannot be exceeded); depot constraints (after a working period, each vehicle returns to the depot from which it departed); pairing constraints
(for a given request, the same vehicle must visit the pick-up and delivery stops); **precedence constraints** (each client must be picked up before being delivered); **availability constraints** (the availability of driver pieces of work must be respected).

**2 Literature**

The only known exact algorithm for the multiple vehicle routing problem with both pick-up and delivery in the presence of time window constraints has been developed by Dumas [12]. This column generation algorithm has been applied to small problems (up to 50 requests) in the context of goods transportation with tight vehicle capacity constraints.

Bodin et al. [3] in their state of the art article on vehicle routing and scheduling problems mention three algorithms for the multiple vehicle dial-a-ride problem. The NEIGUT/NBS procedure schedules one vehicle at a time and determines the best customer to be inserted into the route. The Jaw et al. [13] procedure is a concurrent procedure. The day is split a priori into time slices of 20-30 minutes in length. Assuming that partial vehicle schedules exist over all preceding periods, this procedure forms clusters of customers for each of the vehicles in the given period. The Bodin and Sexton [4] procedure is a traditional “cluster first, route second” approach. For a fixed fleet size, it partitions the set of requests into vehicle clusters and solves the resulting single vehicle dial-a-ride problems using a heuristic based on Benders’ decomposition (Sexton and Bodin [22], [23]). Another concurrent insertion procedure using proximity measures has been developed by Roy et al. [19], [20].

The single vehicle dial-a-ride problem with time windows has also been studied by Psaraftis [17] and Desrosiers, Dumas and Soumis [8]. The interested reader should also refer to the work of Stein [28], Armstrong and Garfinkel [1], and Psaraftis [16], [18] on single and multiple vehicle dial-a-ride problems, to Desrosiers et al. [10], Kolen et al. [14], Savelsbergh [21], Sorensen [27], Solomon [24], [25], Swersey and Ballard [29], and finally to the recent surveys of Desrochers et al. [7] and Solomon and Desrosiers [26] for other variants of vehicle routing problem with time window constraints.

**3 Proposed Methodology**

In a methodology of the “cluster first, route second” type (see Bodin et al. [3]), a cluster is formed from the set of customers assigned to a vehicle, and routing is carried out separately for each vehicle. The routing problems are usually fairly easy to solve. However, the most important decisions are taken at the clustering stage and it is very difficult to globally construct a good set of clusters. For the multiple vehicle dial-a-ride problem, we therefore propose a “cluster first, route second” approach while moving a part of the clustering problem into the routing problem.

At the first stage, rather than defining clusters corresponding to a full day’s work for one vehicle, we define **mini-clusters** grouping together nearby customers who can be transported by the same vehicle over a route segment. This grouping into mini-clusters of similar requests deals with local temporal and spatial considerations only. At the second stage, routes for all the vehicles are constructed