The urban transit crew scheduling problem arises in mass transit corporations who have to create minimal cost bus driver schedule respecting both the collective agreement and the bus schedule. This problem is often modeled as a set covering problem where each column represents a driver’s workday and each row represents a task i.e. a given bus trip. In previous solution methods, a subset of the feasible workdays was created and a heuristic solution using only these workdays was found by solving the resulting set covering problem. We propose a column generation approach to solve the transit crew scheduling problem. The column generation approach decomposes the problem in two parts. The set covering problem chooses a schedule from already known feasible workdays. The subproblem is modeled as a shortest path problem with resource constraints and proposes new feasible workdays to improve the actual solution of the set covering problem. The approach was tested successfully on real-life problems.

1 Introduction

Three main types of algorithms to solve the urban transit crew scheduling problem have been developed and used in practice: the Runcutting, the HASTUS and the Set Covering methods. In this paper, we will restrict ourself to the solution of the Set Covering method using a column generation approach.

The bus schedule defines the vehicle blocks, each vehicle block (or simply block) being a trip starting at the depot and going back to the depot. Along a block, there are relief points. The portion of a block comprised between two consecutive relief points is called a task. For a complete description of the vocabulary used in urban transit crew scheduling, the reader is referred to the glossary by Hartley [3].

The collective agreement and the internal regulations define all the details relevant to the feasible workdays. A workday consist of one or more workpieces executed by the same driver. In the cases where a workday is composed of more than one workpiece, breaks i.e. unworked periods, are inserted between the workpieces. The feasibility of a workday depends on the length of the workpieces and breaks. It is also restricted by conditions on the number of workpieces, the total worked time, the paid time (cost) or the total spread i.e. the elapsed time between the start and the end of a workday. However the feasibility of a workday is not the only aspect regulated by contractual agreement, they also define the cost of all workdays, classify the workdays into types and define constraints on the global schedule.
In Section 2, the new column generation approach to transit crew scheduling is outlined. Computational results on real-life problems are briefly described in Section 3.

2 Crew Scheduling Using a Column Generation Approach

The Set Covering method uses a set covering problem (1-4) to choose the set of variables \( x_j \) each one representing feasible workdays, covering all the rows (tasks) in order to obtain the minimal cost schedule. The covering constraints (2) guarantee that each task \( i \) will be covered by at least one workday \( x_j \), \( a_{ij} \) is equal to 1 if the workday \( x_j \) covers the task \( i \), and to 0 otherwise. The additional constraints (3) usually restrict the proportion of workdays of a given type in the schedule.

\[
\begin{align*}
\text{Min} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j \geq 1 \quad i = 1, \ldots, m \quad (2) \\
& \sum_{j=1}^{n} d_{kj} x_j \leq D_k \quad k = 1, \ldots, K \quad (3) \\
& x_j \in \{0, 1\}. \quad (4)
\end{align*}
\]

Of the three methods mentioned (Runcutting, HASTUS and Set Covering), only the set covering can yield an optimal solution if all the columns are considered during the solution process. So far all the authors have only considered a subset of the feasible workday. However if there is little hope of explicitly considering all the feasible columns in a set covering problem, it is possible to implicitly consider all the feasible workdays in the set covering problem (1-4) and to obtain optimal or near-optimal schedules by using column generation to create crew schedules.

In this section we outline the column generation approach, the modeling of the subproblem and the branching rule used in the Branch & Bound scheme.

2.1 The Column Generation Approach

The column generation approach is used to solve linear programming problems with many columns. In our case, these problems are large set covering problems and are such that a column represents a workday and that each row represents a task. To apply column generation, we must assume that there exist a subproblem to find the minimum marginal cost column.

These set covering problems have too many columns to be handled explicitly. Instead using only a subset \( N^I \) of known feasible routes, the linear programming relaxation of the set covering problem is solved using the simplex algorithm. When the simplex algorithm does the pricing of the variables to