V. Whence the Law of Moment of Momentum?

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1. The Standard Law of Moment of Momentum for Physicists

While mechanics is vulgarly thought a part of physics, it is not through the efforts of physicists that the last two decades have seen a new flourishing of classical mechanics, much in applications, indeed, but even more in basic theory, presented in half a dozen expressly founded journals and a floodlet of books. To the typical physicist, in natural philosophy classical mechanics seems a finished chapter, yet nearly nothing of what he learned about it in the hasty introduction included in his training in theoretical physics is considered generally correct, let alone well done, by the modern theorist of pure mechanics.

The principle of moment of momentum illuminates a fork in method and standpoint as blunt as any professional demarcation today. Let us recall how the principle is presented in a typical textbook of physics. For definiteness, we choose
Joos's *Theoretical Physics*, but nearly any other would serve as well, except that the more recent are often less careful to state explicitly what is done. First, in a system of mass-points the equation of motion for the \( k \)th particle is

\[
m_k \ddot{r}_k = F_k + \sum_i F_{ik},
\]

in standard notation. "NEWTON's third law of the equality of action and reaction" is laid down in the form \( F_{ki} = -F_{hi} \).

Forming \( \dot{H} \), where the moment of momentum \( H \) is defined as

\[
H = \sum_k r_k \times m_k \dot{r}_k,
\]

shows that

\[
\dot{H} = L + \frac{1}{2} \sum_{k,i} (r_k - r_i) \times F_{jk},
\]

where \( L \) is the total torque exerted by the external forces:

\[
L = \sum_k r_k \times F_k.
\]

If the mutual forces \( F_{jk} \) are central, so that \( (r_k - r_j) \times F_{jk} = 0 \), then (3) reduces to

\[
\dot{H} = L,
\]

the desired law. Joos states the result as follows: "*For a system of particles in which the forces between any two particles are in the direction of the line joining these particles, the rate of change of the total angular momentum is equal to the sum of the moments of the applied forces.*" No one can criticize this statement or its proof. However, Joos is uneasy about the relevance of the result, for he adds in fine print:

"The limitation made above is actually of little importance. From considerations of symmetry, it is difficult to imagine a force acting between two points which does not coincide in direction with the line joining them, for there is no other preeminent direction. If the Law of Biot-Savart (p. 290) seems an exception, it must be remembered that this law deals with the force between a magnet-pole and an elementary segment (i.e., not a point or particle) of an electrical conductor."

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