Valuation of the Embedded Prepayment Option of Mortgage-Backed Securities

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1 Introduction

Mortgage-backed securities (MBS for short) combine features of both bonds and options. The investor in the MBS has purchased a fully amortizing bond, with known coupon rate and maturity date. However, at the same time the investor has written a call option to the mortgage borrower. The homeowner's privilege to refinance his or her mortgage at any point in time complicates attempts to price the MBS. Further complications arise due to the homeowners' propensity to prepay their mortgage, even if this decision may not appear to be economically desirable. For example, a homeowner may sell or refinance the house in an unfavorable interest rate environment. Reasons for this decision include, for example, family dynamics (birth, death, divorce) and employment opportunities (relocation).

Traditionally, the pricing of MBS has concentrated on two key aspects of the problem. First, in estimating the prepayment activity of homeowners based on historical observations. Second, in coupling the payout function implied by the prepayment model with appropriate assumptions about the shape of the term structure of interest rate to obtain a fair price for the MBS. Green and Shoven [1986], Richard and Roll [1989] and Kang and Zenios [1992a] are some of the recent publications in the first direction. (See also references therein.) Dunn and McConnell [1981], Brennan and Schwartz [1985], Schwartz and Torous [1989], McConnel and Singh [1990]

1 Special tanks to prof. Marida Bertocchi for her contribution with thwe soluion methodolgy of the differential equation
are some of the publications in the second direction. Recent pricing literature explicitly recognizes that prepayments are exercised in a "suboptimal" fashion, but it does so by assuming some idealistic proportional hazards model to describe prepayment. In contrast, practical applications of MBS pricing utilize elaborate forecasting models for capturing the prepayment activity as a factor of economic factors, seasonality, history of interest rates, demographic considerations, housing activity and so on. See, for example, Hayre [1990] and Hutchinson and Zenios [1991]. Furthermore, practical applications realize the limitations of the prepayment models and the markets desire to be compensated for the additional risk, and resort to option adjusted analysis to compile a rich/cheap profile of MBS in comparison to the Treasuries market. See, again, Hayre and Hutchinson and Zenios.

The prepayment feature of mortgage backed security has always been considered as an essential component for the valuation of MBS. The influence that the prepayment option exerts on MBS pricing has been analyzed by Dunn and McConnel [1981], the prepayment function has been estimated by Schwartz and Torous [1989] and integrated in the MBS valuation, in this paper we try to valuate the prepayment option embedded in the MBS according to specified assumptions.

With the recent interest in the development of analytic models for managing portfolios of MBS — see Hiller and Eckstein [1991] and Zenios [1991] — the temporal variation of the price of the embedded option becomes particularly important. When viewed in the context of a multi-period, dynamic portfolio management problem, one needs to place a price on the MBS at some future time periods, and under different interest rate scenarios. Zenios and McKendall [1991] have developed such models that were subsequently used in constructing mean-absolute deviation efficient portfolios by Kang and Zenios [1992b]. The pricing model of Zenios and McKendall requires the estimation of the option adjusted spread at future points in time. Unfortunately, this spread is obtained empirically based on market data, and hence the pricing model is rather circular. However, if we can obtain the temporal variation of the price of the embedded option (assuming it is exercised optimally) then we can calculate a spread due to the suboptimal exercise of the option. This spread will presumably remain constant across time, and can be calibrated based on today's prices. It can then be used to price the MBS at some point in the future. In essence, we are separating the MBS into a fully amortizing bond, a call option with optimal exercise policies, and a spread due to the additional risks created by the stylized behavior of homeowners. The last component is constant across time, and for the first two we can obtain pricing equations based on standard financial theories.

The purpose of this paper is precisely to develop a pricing model for the embedded prepayment option. The theory of option pricing has had extensive applications to different contingent claims, as well as the pricing of callable warrants, see Merton [1973], or the pricing of convertible saving bonds, Brennan and Schwartz [1979a,1982] and callable bonds. We apply options pricing theory to value the prepayment option embedded in a MBS due to changes in interest rates, ignoring the influence exerted by other factors. We use the seminal model of Black and Scholes [1973] as extended by Merton [1973], and couple those with the interest rate model of Brennan and Schwartz [1983]. Hence we derive the stochastic dif-