Reflection of a Plastic Wave at an Obstacle

By N. V. Zvolinsky and O. V. Rykov
Institute of Geophysics, Moscow, U.S.S.R.

1. Introduction

The propagation of a plastic wave and its interaction with an obstacle have been investigated by several authors. References [2] and [3] are concerned with the propagation, and references [4] and [5] with the interaction problem. In these papers it was assumed that the density of the material remained constant during unloading and linear or piece-wise linear approximations were used to the laws of loading.

The present paper shows that the interaction of a plane wave with an obstacle is readily treated when the stress during loading is supposed to be proportional to a power of the strain. A power law of this kind is in good agreement with experiments over a wide range of stress.

The results of the present investigation are useful in the treatment of wave phenomena in soft soil.

2. Preliminary Remarks

For the study of plane waves, the properties of the material are adequately specified by two pieces of information, namely

(a) the law of cubical compression

\[ \sigma = \sigma(\theta) \]  \hspace{1cm} (2.1)

and

(b) the yield condition

\[ |\sigma_x - \sigma_y| = -m\sigma + m'. \]  \hspace{1cm} (2.2)

Here, \( \sigma_x \) and \( \sigma_y \) are the extreme principal stresses, \( \sigma \) is the mean normal stress, \( \theta \) the cubical compression, and \( m \) and \( m' \) are positive constants. Note that only phenomena involving plastic deformation are discussed in this paper.

The law of cubical compression is assumed to be different during loading (line AB in Fig. 1) and unloading (line BD): the mean normal
stress $\sigma(\theta)$ is given by $\sigma(\theta) = f_1(\theta)$ for loading ($d\sigma/dt > 0$), while it is indeterminate for unloading ($d\sigma/dt < 0$).

If we restrict ourselves to the discussion of a single plane plastic wave, we only need the uniaxial equivalent of (2.1) and (2.2), which we assume to be

$$\sigma_x = \sigma^0 |\varepsilon_x|^n, \quad \sigma^0 < 0, \quad n > 1, \quad (2.3)$$

where $\sigma^0$ and $n$ are constants.

In agreement with experimental facts, we assume that on reloading (after unloading of the type represented by the line $BD$ in Fig. 1) the increase of $\sigma_x$ is not accompanied by a change in density until $\sigma_x$ has reached the value (represented by the ordinate of $B$) at the beginning of the last unloading process. The behavior of the material under further loading is represented by the continuation $BC$ of the original loading diagram.

For simplicity, the index $x$ will be omitted in the following, and $\sigma$ and $\varepsilon$ will be used instead of $\sigma_x$ and $\varepsilon_x$. As we only intend to discuss plastic phenomena, we may use the law (2.3) which disregards an initial elastic part of the loading diagram.

3. Propagation of a Plane Plastic Wave Generated by Exterior Action

Let the uniform stress at a certain material plane be given as a function of time. Waves will propagate from this plane in both normal directions. We consider one of these waves and let the $x$ axis be parallel to the line of propagation. The phenomenon is conveniently described by using the Lagrangian coordinate $h$ and the time $t$ as independent variables and writing the Eulerian coordinate $x(h, t)$ in the form $x(h, t) = h + u(h, t)$, where $u(h, t)$ is the displacement of the particle $h$.

We take $u(h, 0) = 0$, that is, we suppose that the medium is initially unperturbed. At the material plane $h_0$, the stress $\sigma_0$ is prescribed as a function of time; its absolute value is assumed to increase instantaneously at $t = 0$ and then to decrease. These assumptions concerning the character of the excitation are essential for the following treatment. It is natural to expect that the instantaneous loading produces a shock front. Let $h^*$ be the Lagrangian coordinate of this front (Fig. 2). Particles with Lagrangian coordinates $h$ satisfying $h^* < h < h_0$ are in a region