The aim of my talk is to present some basic results from the domain of Combinatorial Integral Geometry concerning lines and planes in three-dimensional Euclidean space $\mathbb{R}^3$.

The subject of Combinatorial Integral Geometry presumably has direct applications in Mathematical Stereology. It is the topic of a special book $<1>$ where among other things complete proofs of the statements we are going to formulate can be found.

The corresponding planar results were the topic of my lecture delivered at the first Buffon Bicentenary meeting which was held in Erevan (Sevan) last autumn. Results in three dimensions have since been derived starting from the planar ones. The complete account of the present state of the planar theory will soon be published $<2>$ in a form accessible to most listeners today. Also three dimensions is the case which is of more interest from the point of view of applications. Therefore I choose to describe here the available results for $\mathbb{R}^3$, giving only minimal reference to the more extensively studied planar case.

I - BUFFONIC ROOTS

Let us reformulate the classical Buffon's problem of the needle as follows.

Instead of throwing the needle at random onto a floor ruled with a lattice of equidistant parallel lines, rather suppose that a needle $\nu$ of length $|\nu| < 1$ is fixed (say in a disc $K$ of unit diameter) and the lattice of parallel lines with unit separation is randomly placed on the plane. What is the probability that the line of the lattice will hit the needle?

The position of the lattice is completely specified by that of its almost surely unique member line $g_k$ intersecting $K$. For this reason to define the distribution of the position of the lattice it suffices to determine a distribution of a random line through $K$ (that is of $g_k$). In these terms the "problem of the needle" is equivalent to the problem of finding the probability of the event...
"g_k belongs to the set [v]."

Here [v] is the subset of the space G of lines on the plane defined as

\[ [v] = \{ g \in G : \text{the line } f \text{ intersects the needle } v \} \cdot \]

Thus we maintain that 200 years ago Buffon was calculating the probability (or measure) of the set [v]. For this reason the subsets of G of the type [v] have been called in \(< 2>\) Basic Buffon.

Note that the measure on G which corresponds to the original Buffon's needle-throwing experiment is in fact proportional to the so-called invariant measure on G.

In three dimensions there are two analogues of the classical Buffon problem of the needle.

I - A lattice of equidistant parallel planes with unit separation is fixed in \( \mathbb{R}^3 \) and a needle \( n \) of length \( |n| < 1 \) in placed at random in \( \mathbb{R}^3 \). What is the probability that \( n \) will hit a plane belonging to the lattice?

II - In \( \mathbb{R}^3 \) a lattice of lines parallel to the Oz axis is fixed. For instance we may assume that the points of intersection of the lines from the lattice form a square lattice on the plane \( z = 0 \). A flat \( f \) (a flat is defined as a bounded convex part of a plane) with maximal diameter less then 1 is placed at random in \( \mathbb{R}^3 \). What is the probability that \( f \) will hit a line from the lattice?

These two problems also permit reformulations in which it is the position of the lattice which is assumed random. Thus equivalent problems I' and II' arise:

I' - Let \( E \) be the space of all planes in \( \mathbb{R}^3 \). Define the subset \( [n] \subset E \) to be the set of all planes which intersect the needle \( n \subset \mathbb{R}^3 \). Find the measure (probability) of the set \( [n] \).

II' - Let \( \Gamma \) be the space of all lines in \( \mathbb{R}^3 \). Define the subset \( [f] \subset \Gamma \) to be the set of all lines intersecting the flat \( f \). Find the measure (probability) of the set \( [f] \).