Superconductivity, one of the most fascinating phenomena in solid-state physics, was discovered in 1911 by Kamerlingh Onnes [14.1], but it was not until 1957 that a satisfactory microscopic theory of the effect became available, i.e., the one by Bardeen, Cooper and Schrieffer (BCS) [14.2]. The major obstacle theorists were faced with earlier can be summarized as follows. The superconducting transition temperature $T_c$ is usually of the order of 10 K, which corresponds to an energy of order 1 meV (we are not considering for the moment the more recent high-temperature superconductors). Provided that superconductivity is based on electron correlations and taking into account that the correlation energy of electrons in a metal is of the order of 1 eV per electron, is it then necessary to compute that energy to an accuracy of order 1% to find a superconducting ground state? This would indeed be an impossible task and eliminate any hope for a microscopic theory. Fortunately, a very special correlation leads to the phenomenon of superconductivity and the treatment of the remaining correlation contributions is unnecessary. All the correlations that are difficult to treat—and thus have been left out—enter the theory only in the form of renormalization parameters. An example are the heavy-fermion systems, in which the characteristic strong correlations manifest themselves in the form of quasiparticle energies; the (pair) correlations responsible for superconductivity are added and treated separately. This explains why reliable calculations of the superconducting transition temperature have so far remained an unsolved problem. They would require a microscopic calculation of those parameters.

The special correlations responsible for superconductivity are pair correlations. In the presence of electron attractions they lead to the formation of electron pairs (Cooper pairs) [14.3]. In principle, pairs may also form when the electron interactions are purely repulsive, but then they must meet certain stringent requirements. For example, the interaction must be much less repulsive for electrons near the Fermi surface than away from it. Another important finding is that electron pairs can be treated as being independent of each other, a natural generalization of the concept of independent single electrons. The ground-state wavefunction has therefore the form of an antisymmetrized product of pair wavefunctions. If we are to do actual calculations with such a wavefunction, it has to be written in the form of a coherent state of, pairs of electrons. Since there are a number of excellent textbooks available on the
theory of superconductivity [14.4–7], we have kept the following discussion relatively short and condensed. We cover here only those aspects of superconductivity required to obtain a balanced, overall view of the effects of electron correlations in solids.

The field of superconductivity received an immense impetus from the discovery of the new high-temperature superconducting materials in 1986 by Bednorz and Müller [14.8]. The subsequent development has raised the transition temperature \( T_c \) to values as high as \( T_c = 125 \) K. Examples of the new high-\( T_c \) materials are \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) \((T_c \approx 40 \) K\), \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) \((T_c = 92 \) K\), \( \text{Bi}_2\text{Sr}_2\text{Ca}_x\text{Cu}_3\text{O}_{10} \) \((T_c = 110 \) K\), and \( \text{Tl}_2\text{Ca}_2\text{Ba}_2\text{Cu}_3\text{O}_{10} \) \((T_c \approx 125 \) K\). Whereas in these systems conduction is due to holes, in \( \text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4 \) \((T_c \approx 20 \) K\)—an example of an electron-doped system—this is different. An important property of these materials is the strength of their electron correlations. We refer in particular to the copper-oxide planes in which superconductivity takes place. Research on the high-\( T_c \) materials has to include the strong correlations and to provide ways of dealing with them. It has been argued that the strong correlations provide for the electron attractions which are required in order to obtain the high superconducting transition temperatures in the copper-oxide based materials [14.9], but we know also that the electron–phonon interaction is strong in those systems and therefore must contribute to \( T_c \) significantly. As in any other rapidly developing field of research, many speculations and suggestions have been advanced. We will restrict ourselves to a discussion of only those aspects of the strong correlations in the high-\( T_c \) materials that seem to be well understood already.

### 14.1 The Superconducting State

The Cooper instability that a system of normal electrons may experience [14.3] constitutes the key to the phenomenon of superconductivity. Consider a filled Fermi sphere in momentum space with radius \( k_F \) and two extra electrons outside of it (Fig. 14.1). These two electrons are assumed to attract each other through a potential \( V(r_1 - r_2) \). The center of mass is assumed to be at rest. None of the

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**Fig. 14.1.** Two interacting electrons outside a filled Fermi sphere. When the interaction is attractive they form a bound state (Cooper pair)