5. Joint Space Decomposition Approach

In this section a separation method for minimum-time trajectory planning for serial redundant robots with one redundant degree of freedom is presented. As seen in Chapter 4, the path in space is parametrized by a scalar parameter $s$. The approach is based on [9] where the robot joints are divided in two sets, the non-redundant of the same dimension as the task space and the remaining redundant joint. Time-optimal trajectories for the task space path parameter $s$ and for the redundant coordinate $q_r$ are obtained by means of an optimization problem. The non-redundant joint positions are found using analytic inverse kinematics for the end-effector position computed by means of the parameterized path and the position of the redundant joint.

In the present thesis time-parametrized multi-interval B-spline curves are used expressing the trajectories of the task space path parameter $s$ and the position of the redundant degree of freedom $q_r$. The end time and control points of the curves are used as optimization parameters allowing to incorporate technological and physical constraints of the robot and its environment. Moreover, an approach to reduce the complexity of finding initial values for the optimization process will be presented.
Joint Space Decomposition Approach

5.1 Method

In Section 4.3 a B-spline curve is used to parametrize the path parameter \( s \) of a task space path \( \mathbf{r}_{E,d} \) and numeric inverse kinematic approaches are used to compute the joint trajectories \( q_i(t) \). The present approach adopts the parametrization of the end-effector position path \( \mathbf{r}_{E,d} \), i.e.

\[
\mathbf{r}_{E,d} = \mathbf{r}_{E,d}(s)
\]

with

\[
s(t) = \sum_{j=0}^{m_s-n_s-2} d_{s,j} N_{s,j}^{n_s}(t), \quad t \in [0, t_E],
\]

where \( m_s \) denotes the number of knots and \( n_s \) is the maximum degree of \( s(t) \). \( t_E \) is the end time of the trajectory, \( d_{s,j} \) are the B-spline control points and \( N_j^{n_s} \) are the B-spline basis functions.

Similar to [9], the manipulator’s joints are separated in one freely selected redundant joint \( q_r \) and a non-redundant part \( \mathbf{q}_{nr} \) that consists of the rest of the joints, i.e.

\[
\mathbf{q}_{nr} = \mathbf{S}_{nr} \mathbf{q}
\]

\[
q_r = \mathbf{s}_r^\top \mathbf{q}
\]

where the selection mask matrix \( \mathbf{S}_{nr} \) and the selection mask vector \( \mathbf{s}_r \) perform the separation mentioned above.

The separation process will be further clarified with the following example: If the vector of joint coordinates is \( \mathbf{q} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^\top \) and it is chosen to treat \( q_3 \) as the redundant degree of freedom, the selection masks \( \mathbf{S}_{nr} \) and \( \mathbf{s}_r \) are found to be

\[
\mathbf{S}_{nr} = \text{diag}(1, 1, 0), \quad \mathbf{s}_r = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top.
\]

For the trajectory of the redundant degree of freedom \( q_r \) an additional B-spline curve is assumed, i.e.

\[
q_r(t) = \sum_{j=0}^{m_s-n_s-2} d_{r,j} N_{r,j}^{n_r}(t), \quad t \in [0, t_E],
\]