RANDOM BEHAVIOR IN NUMERICAL ANALYSIS, DECISION THEORY, AND MACROSYSTEMS: SOME IMPOSSIBILITY THEOREMS

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Abstract: For many topics, including decision analysis, policy making, and the normative study of certain macrosystems, tools of analysis are applied to determine the essence or the state of a problem. The one commonality among these tools is that we want them to be "reliable". For certain standard tools, it is shown that this goal of reliability may be impossible to attain. For some of these impossibility statements, alternative approaches are suggested.

1. Introduction

Certain basic tools are commonly used both with decision analysis and with macrosystems. Some of these tools are devices designed to be incorporated within the system in order to assist and to influence the subsequent dynamics. For instance, this includes any method used to facilitate the decision making within an organization. Here an obvious example would be voting methods used to aggregate individual differing rankings over several alternatives into one common group ranking. Other types of tools are the techniques used in systems analysis. On a theoretical level, this may be an algorithm designed to seek a zero of a smooth function - such a zero may correspond to an equilibrium or an optimal point for a system. Or, it may be an integer programming problem used to determine an efficient policy. It may be the statistical and probabilistic tools developed to understand and to interpret data - perhaps to aid in a decision analysis or a policy decision.

Central to the selection of any tool is the requirement that it is reliable. Here there are at least two criteria. First of all, the tool should apply for all of the situations within a class of interest; that is, we seek <u>universal</u> <u>mechanisms</u>. For instance, when we search for a zero or an equilibria of a function g, we prefer to use an algorithm which always will work as long as g is sufficiently smooth. Indeed, this is part of the historical attraction of the tatonnement story from economics; it has been viewed as being a universal mechanism where the market forces of supply and demand iteratively converge to a market equilbrium price.

A mechanism or tool is selected to achieve a specified goal. Consequently, a second crucial condition is that the tool doesn't lead to unexpected surprises, conclusions, or consequences which may violate or vitiate our objectives; we want <u>the</u> <u>outcomes</u> <u>to</u> <u>consistently</u> <u>reflect</u> <u>these</u>

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objectives. For instance, in the choice of a voting method, we want the final result to accurately reflect the preferences of the electorate. As a hypothetical example, consider the problem of selecting a common beverage for lunch where a voting method is used to guide us in the decision process. Suppose a vote leads to the ordering wine > water > beer. Should wine be unavailable, we would expect to be able to replace it with the second ranked choice of water free of fear that a majority of the people really would have preferred beer.

An impossibility theorem arises when certain basic objectives are frustrated; when there doesn't exist a devise or a mechanism which satisfies the specified criteria. Therefore, the theme of this paper which is that impossibility theorems play an important role in the system sciences, is somewhat disturbing. Often such theorems arise because mechanisms violate conditions which are "intuitively obvious"; in this setting, an impossibility statement is called a "paradox".

In this paper, I'll consider several paradoxes and impossibility theorems with three goals in mind. The first is to introduce several new impossibility theorems related to the topics mentioned above. The second is to take these seemingly disparate results and to unify them by showing that they have a common explanation. (Although I will not develop the theme here, this approach relates these new results to several important paradoxes such as Arrow's Theorem, the Alabama paradox of apportionment, etc.) Finally, I'll briefly note some research, still in its infancy, which has the goal either to circumvent, or to handle the disturbing consequences of these impossibility results.

2.The source of the problem

All of the results to be discussed here are caused by the inverse image of certain functions being multivalued in a particular manner. To understand the basic idea, consider the function f represented in Figure 1.

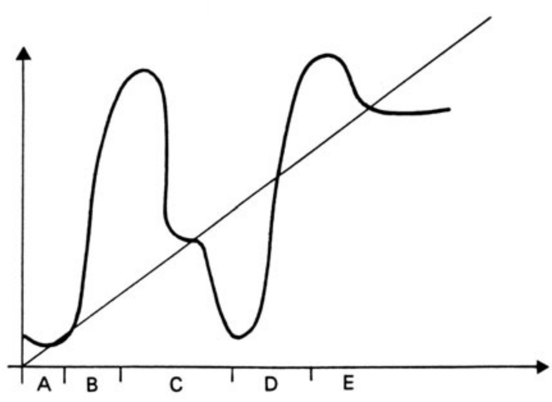

FIGURE 1

The inverse function, f^{-1}, clearly is multi-valued; indeed, in this figure, the intervals A, B, C, D, E designate those regions over which f^{-1} is single valued. To see what mischief this multivalued property can create, consider the trajectories of the deterministic system

2.1) $\quad x_{N+1} = f(x_N).$