HEAVY VIABLE TRAJECTORIES OF CONTROLLED SYSTEMS

Jean-Pierre Aubin and Halina Frankowska
CEREMADE, University of Paris-Dauphine, Paris, France

ABSTRACT

We define and study the concept of heavy viable trajectories of a controlled system with feedbacks. Viable trajectories are trajectories satisfying at each instant given constraints on the state. The controls regulating viable trajectories evolve according a set-valued feedback map. Heavy viable trajectories are the ones which are associated to the controls in the feedback map whose velocity has at each instant the minimal norm. We construct the differential equation governing the evolution of the controls associated to heavy viable trajectories and we prove their existence. These results are applied to exchange economies for finding heavy trajectories of a dynamical decentralized allocation mechanism explaining the evolution of prices.

1. INTRODUCTION

When we study the evolution of macrosystems which arise in economics and the social sciences as well as in biological evolution, we should take into account not only :

(1) our ignorance of the future environment of the system

but also :

(2) the absence of determinism (including the impossibility of a comprehensive description of the dynamics of the system)
(3) our ignorance of the laws relating certain controls to the states of this system
(4) the variety of dynamics available to the system.

We propose to translate these requirements into mathematics by means of differential inclusions, which describe how the velocity depends in a multi-valued way upon the current state of the system. Another feature of such macrosystems is that the state of the system must obey given restrictions known as viability constraints, which determine the viability domain ; viable trajectories are those lying entirely within the viability domain. Finding viable trajectories of a differential inclusion provides a mechanism of selection of trajectories which, contrary to optimal control theory, does not assume implicitly

(1) the existence of a decision maker operating the controls of the system (there may be more than one decisionmaker in a game-theoretical setting)

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(2) the availability of information (deterministic or stochastic) on the future of the system ; this is necessary to define the costs associated with the trajectories

(3) that decisions (even if they are conditional) are taken once and for all at the initial time.

Viability Theorems provide necessary and sufficient conditions for the existence of at least one viable trajectory starting from any viable initial state. It also provides the feedbacks (concealed in both the dynamics and the viability constraints) which relate the state of the system to the controls. These feedbacks are not necessarily deterministic : they are set-valued maps associating a subset of controls with each state of the system. We observe that the larger these subsets of controls are, the more flexible - and, thus, the more robust - the regulation of the system will be.

Finally the third feature shared by those macrosystems is the high inertia of the controls which change only when the viability of the system is at stake. Associated trajectories are called heavy viable trajectories : they minimize at each instant the norm of the velocity of the control. We shall provide a formal definition of heavy viable trajectories, which requires an adequate concept of derivative of the set-valued feedback map. We show that as long as the state of the system lies in the interior of the viability domain, any regulating control will work. Therefore, along a heavy trajectory, the system can maintain the control inherited from the past. (The regulatory control remains constant even though the state may evolve quite rapidly).

What happens when the state reaches the boundary of the viability domain ? If the chosen velocity is "inward" in the sense that it pushes the trajectory back into the domain, then we can still keep the same regulatory control.

However, if the chosen velocity is "outward", we are in a period of crisis and must find, as slowly as possible, another regulatory control such that the new associated velocity pushes the trajectory back into the viability domain.

When this strategy for "structural change" fail, the trajectory "dies" i.e., it is no longer viable (see Figure below).

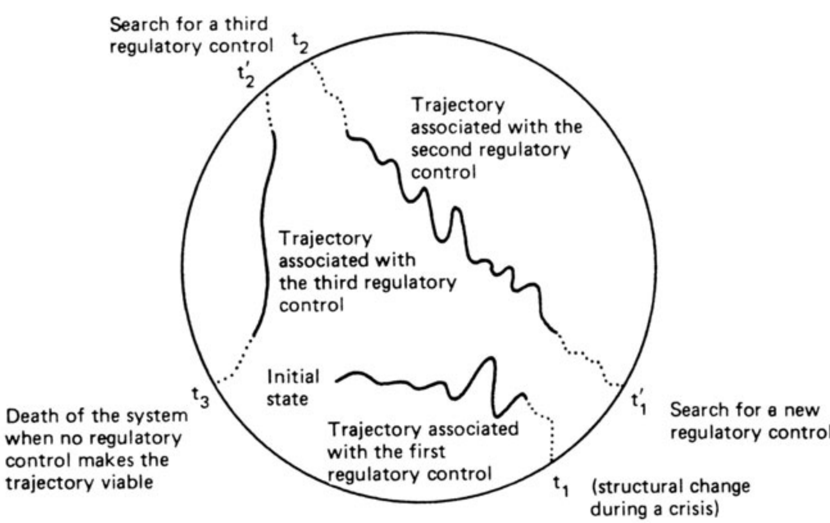

FIGURE 1 The viability domain