Elastic Columns, plates, shells and some other structures can collapse at a modest stress level, due to an instability of the equilibrium. Viscoelastic structures display similar, but more complex phenomena, which we now shall study for the case of a simple column.

7.1 The Concept of Stability

Two definitions of stability are in common use:

First Definition – When a system in equilibrium is subjected to a small (infinitesimal) disturbance, it may happen that it returns to its original position when the disturbance is removed. In this case the system is in stable equilibrium.

If the system is endowed with mass and suddenly unloaded, it will not stop in the equilibrium position, but will vibrate about it. If there is no inertia, the return is immediate.

Second Definition – A system is in stable equilibrium if there does not exist any adjacent position for which its potential energy is smaller.

Both definitions are identical for conservative systems. For a nonconservative system, the second definition becomes meaningless, since a potential energy cannot be defined. But the first definition is also not of much use. A nonconservative system, when disturbed, may never return to its original position (for example, if dry friction or plastic deformation is involved), but if a small disturbance causes only a small
displacement, it is for practical purposes as safe as a stable conservative system. Therefore the stability concept will be avoided altogether in the study of the behavior of viscoelastic columns.

7.2 Inverted Pendulum

Before approaching the column problem, let us have a look at a simpler problem of the same kind, the inverted pendulum shown in Fig. 7.1. A rigid, massless, and weightless bar is supported by a frictionless hinge and carries at its upper end a rigid body of weight P. This body does, necessarily, have a mass P/g, but in true stability problems mass is irrelevant, and in the slow motion of viscoelastic creep its influence is negligibly small.

In Fig. 7.1a the pendulum is braced laterally by a spring of stiffness k. In the undisturbed state the pendulum is vertical and the spring is undeformed. We disturb it by tilting it slightly as shown. This extends the spring by $u = \theta l$ and produces a spring force $F = ku$. The forces P and F have moments with respect to the hinge, and the resultant moment, positive when clockwise, is

$$M = Pl\theta - Fl = (P - kl)u. \quad (7.1)$$