1 Classification of Scheduling Problems

The theory of scheduling is characterized by a virtually unlimited number of problem types. In this chapter, a basic classification for the scheduling problems covered in the first part of this book will be given. This classification is based on a classification scheme widely used in the literature (see e.g. Lawler et al. [1993]). In later chapters we will extend this classification scheme.

1.1 Scheduling Problems

Suppose that \( m \) machines \( M_j (j = 1, \ldots, m) \) have to process \( n \) jobs \( J_i (i = 1, \ldots, n) \). A schedule is for each job an allocation of one or more time intervals on one or more machines. Schedules may be represented by Gantt charts as shown in Figure 1. Gantt charts may be machine oriented (Fig. 1(a)) or job oriented (Fig. 1(b)).

Figure 1.1
1.2 Job Data

A job $J_i$ consists of a number $n_i$ of operations $O_{i1}, \ldots, O_{in_i}$. Associated with operation $O_{ij}$ is a processing requirement $p_{ij}$. If job $J_i$ consists only of one operation ($n_i = 1$) then we identify $J_i$ with $O_{i1}$ and denote the processing requirement by $p_i$. Furthermore, a release date $r_i$, on which the first operation of $J_i$ becomes available for processing may be specified. Associated with each operation $O_{ij}$ is a set of machines $\mu_{ij} \subseteq \{M_1, \ldots, M_m\}$. $O_{ij}$ may be processed on any of the machines in $\mu_{ij}$. Usually, all $\mu_{ij}$ are one element sets or all $\mu_{ij}$ are equal to the set of all machines. In the first case we have dedicated machines. In the second case the machines are called parallel. The general case is introduced here to cover problems in flexible manufacturing where machines are equipped with different tools. This means that a job can be processed on any machine equipped with the appropriate tool. We call scheduling problems of this type problems with multi-purpose machines ($MPM$).

It is also possible that all machines in the set $\mu_{ij}$ are used simultaneously by $O_{ij}$ during the whole processing period. Scheduling problems of this type are called multiprocessor task scheduling problems. Multiprocessor task scheduling problems and scheduling problems with multi-purpose machines will be classified in more detail in Chapters 10 and 11.

Finally there is a cost function $f_i(t)$ which measures the cost of completing $J_i$ at time $t$. A due date $d_i$ and a weight $w_i$ may be used in defining $f_i$.

In general, all data $p_i, p_{ij}, r_i, d_i, w_i$ are assumed to be integer. A schedule is feasible if no two time intervals on the same machine overlap, if no two time intervals allocated to the same job overlap, and if, in addition, it meets a number of problem-specific characteristics. A schedule is optimal if it minimizes a given optimality criterion.

Sometimes, it is convenient to identify a job $J_i$ by its index $i$. We will use this brief notation in later chapters.

We will discuss a large variety of classes of scheduling problems which differ in their complexity. Also, the algorithms we will develop are quite different for different classes of scheduling problems. Classes of scheduling problems are specified in terms of a three field classification $\alpha|\beta|\gamma$ where