16  The Smoothness of Kriging

How smooth are estimated values from kriging with irregularly spaced data in a
moving neighborhood? By looking at a few typical configurations of data around
nodes of the estimation grid and by building on our knowledge of how spatial
components are kriged, we can understand the way the estimated values are
designed in ordinary kriging.

The sensitivity of kriging to the choice of the variogram model is discussed
in connection with an application on topographic data.

Kriging with irregularly spaced data

Let us take a regionalized variable for which three components of spatial variation
have been identified. For the following linear model of regionalization we assume
local stationarity of order two

\[ Z(x) = Z_0(x) + Z_1(x) + Z_2(x) + m(x) \]

The covariance model of the three spatial components is

\[ C(h) = C_0(h) + C_1(h) + C_2(h) \]

where we let \( C_0(h) \) be a nugget-effect covariance function and \( C_1(h), C_2(h) \) be
spherical models with ranges \( a_1 \) and \( a_2 \), numbered in such a way that \( a_1 < a_2 \).

Suppose that the sampling grid is highly irregular, entailing a very unequal
distribution of data in space. A grid of estimation points is set up, with nodes
as close as is required for the construction of a map at a given resolution. At
each point the operation of ordinary kriging is repeated, picking up the data in
the moving neighborhood.

Assuming a highly irregular arrangement of the data points, very different
configurations of sample points around estimation points will arise, four of which
we shall examine in the following.

1. \( x_0 \) is more than \( a_2 \) away from the data

This configuration of data points around the estimation point can arise when \( x_0 \)
is located amid a zone without data within the range of the two spherical models,
as shown on Figure 16.1. Ordinary kriging is then equivalent to the kriging of
the mean. The right hand side covariances of the ordinary kriging system are nil
as all distances involved are greater than $a_2$. The information transferred to $x_0$
will be an estimation of the local mean in the neighborhood.

This shows that kriging is very conservative: it will only give an estimate of
the mean function when data is far away from the estimation point.

2. $x_0$ is less than $a_2$ and more than $a_1$ away from the nearest data point $x_\alpha$

With this spatial arrangement, see Figure 16.2, the ordinary kriging is equivalent
to a filtering of the components $Z_0(x)$ and $Z_1(x)$, with the equation system

$$
\begin{align*}
\sum_{\beta=1}^{n} \lambda_\beta C(x_\alpha - x_\beta) - \mu &= C_2(x_\alpha - x_0) \\
\sum_{\beta=1}^{n} \lambda_\beta &= 1
\end{align*}
$$

The covariances $C_0(h)$ and $C_1(h)$ are not present in the right hand side for
kriging at such a grid node.

3. $x_0$ is less than $a_1$ away, but does not coincide with the nearest $x_\alpha$

In this situation, shown on Figure 16.3, ordinary kriging will transfer information
not only on the long range component $Z_2(x)$, but also about the short range
component $Z_1(x)$, which varies more quickly in space. This creates a more
detailed description of the regionalized variable in areas of the map where data